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An Optimization Model for Hospitals Inventory Management in Pharmaceutical Supply Chain

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ABSTRACT

Pharmaceutical inventory problem which include the planning of pharmaceutical inventory is considered. The planning is based on the patient population which comes up as uncertain and vary through time. So that the demand becomes stochastic. Therefore, it is necessary to develop information to understand the relationship between patients arrivals, conditions, and demand for the pharmaceutical inventories. In Indonesia, the implementation of e-catalog system still causes some obstacles in terms of medicine availability. The direct appointment of the pharmaceutical factories and wholesalers has resulted production and distribution of drugs which are not optimal yet. What happens is the uncertainty in terms of the time order received (lead time) and the received quantity. Hence, the Hospital pharmacy (HP) must be able to respond to this by developing an inventory policy which are able to provide the sustainable pharmaceutical inventories. The objectives of this paper are to develop a multi-stage stochastic programming model to optimize the pharmaceutical inventory costs with uncertainty in demand, lead time and received quantity. This model assumes a continuous review policy, with a (Q, r) model for multi-products in one echelon pharmaceutical supply chain, i.e. a hospital. Stock out is overcome by backorder entirely, without considering loss sale.

Keywords: pharmaceutical supply chain, pharmaceutical inventory, stochastic programming model, uncertainty, continuous review, multi-products.

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INTRODUCTION

Management of pharmaceutical services is a cycle of activities that includes selection, needs planning, procurement, acceptance, storage, distribution, destruction and withdrawal, control and administration. This management is administered by the Hospital Pharmacy (HP). Pharmaceutical management is needed to minimize waste of resources and mishandling of uses. The HP director must be able to develop inventory policies by considering uncertain demands, limited storage capacity, customer service level (CSL), patient safety and various regulations that may affect supply (Uthayakumar & Priyan, 2013).
Demand for pharmaceutical products (D_i) came from the doctor regarding the decisions on treatment for patients. The doctor will diagnose the disease, and then prescribe the medicine to be used. Pharmaceutical inventory planning is based on a patient population in the hospital that is uncertain and varies over time. As a result, demand becomes stochastic. It is therefore necessary to develop information in order to understand the relationship between the patient's arrival, his condition and the demand for pharmaceutical products.
The health care system is challenged to deliver quality services at affordable prices. However, an increase in the prices of health products and services, including the availability of products and medical treatment, requires hospitals to reduce operating costs without affecting the quality of their services. Budgeted costs for the provision of pharmaceutical services (TC_i) are the largest component of hospital expenditure. In many developing countries, the provision of pharmaceutical services in hospitals can absorb some 40-50 percentages of the total hospital costs. There is no doubt that the provision of such large pharmaceutical services must be managed effectively and efficiently, which is necessary given the need for funding to provide pharmaceutical services in hospitals is not always in line with the request.
The pharmaceutical supply chain consists of the stage of production (drug factory), the stage of distribution (pharmaceutical wholesalers) and the stage of service (hospitals and pharmacies). The pharmaceutical supply chain is very complex and requires a great responsibility to ensure that the right product reaches the right person at the right time and maintains the quality of the product. The pharmaceutical supply chain must therefore fully reach the level of customer service (CSL) because it can directly affect the health and safety of patients. In Indonesia, various regulations have been established to

control prices, drug shortages and the suitability of drug use.

In inventory management, the Economic Order Quantity (EOQ) model defines the order quantity (Q^*) which minimizes inventory costs and total order costs. EOQ is only used when the demand for a product is constant throughout the year and every new order is fully delivered when the inventory is zero ($I(t) = 0$), assuming a constant lead time. Order costs remain constant irrespective of the number of units ordered. The model also takes into account storage costs as a percentage of purchase costs. However, this model does not allow for stock-outs, and demand that varies over time (dynamic) (Uthayakumar & Karuppasamy, 2018). Also, in reality, the number of requests and lead times often varies, and the amount received does not match the order (Bartmann & Beckmann, 1992). In order to overcome these problems, this EOQ model needs to be developed.

The inventory is used to meet patient demand, so it is important to estimate the exact amount and timing of demands in the planning process. It is also important to know how long it will take until the order has been received, so as not to run out of stock (stockout). When demand is stochastic, lead time become the parameter that needs to be reduced. The usual lead time is reduced by paying more (penalty charges, c_s) to the pharmaceutical wholesaler.

Uthayakumar dan Priyan (2013) have developed an inventory model that integrates continuous review with production and distribution of supply chains related to drug factories and hospitals. This model considers multiple products, lead time variables, payment delays, limited storage space, and CSL. This model calculates the minimization of total inventory costs by taking decisions on the optimal order number (Q_i), lead time (L) and number of shipments n for all products in a single cycle. However, this model does not take into consideration uncertainty about demand, lead time and number of received orders.

In addition, Uthayakumar dan Karuppasamy (2018) developed a pharmaceutical inventory model by assuming demand as a quadratic function and storage costs as a linear function, where payment delays were allowed. This model is based on the backorder of some products. Priyan dan Uthayakumar (2014) developed the Uthayakumar dan Priyan (2013) models by adding uncertainty to the number of orders received. However, this model did not consider uncertainty about demand and lead time.

With a variety of uncertain constraints in the real world, stochastic programming is used to model this uncertainty. The most commonly used stochastic programming is two-stage stochastic programming. Cunha et al. (2017) has developed a two-stage stochastic model for a single item in a single row, with uncertain demand. Approximate the solution using a mixed-integer programming model using a limited number of scenarios. However, this model considers a policy of periodic review.

The aim of this study is to develop a multi-stage stochastic programming model to optimize the inventory costs of pharmaceutical products in hospitals in the presence of a number of uncertain factors, including demand, lead time and the number of received orders.

MULTI-STAGE STOCHASTIC PROGRAM MODEL FOR PHARMACEUTICAL INVENTORY

The stochastic program model is a branch of mathematics for situations where the decision contains data uncertainty. This model is used to construct mathematical formulations to determine the optimization of pharmaceutical inventory costs in hospitals with uncertainty in demand, lead time and number of received orders.

Scenario of Inventory Problems in Hospitals

The model is assumed to be three phases, each of which consists of decisions (scenarios) taken by HP, taking into account the overall cost efficiency. And decision-making at each stage depends on the implementation of the previous stage.

In the first stage, the decision is to determine the optimal order quantity (Q_i) and the reorder point (r_i) which minimizes the total inventory cost (TC). The second stage is a decision in the lead time. At this stage, if demand during lead time (X_i) exceeds expectations, where $X_i > r_i$ is expected, there will be a stock-out. This stock-out will be overturned with a fully back-order (not considering a loss sale). Although the number of orders in the first phase has taken into account demand and safety stock expectations, it is still possible to have a stockpiling. In the case of hospitals, it is not possible to leave unfulfilled demands (loss sale). Therefore, the stock-out must be overcome by a fully back-order. Backorder can be done on the same pharmaceutical wholesalers or on different ones. This depends on the level of drug requirements that pharmaceutical wholesalers can meet. Backorder may be received instantly or wait until it is delivered along with the previous order, in this case with an uncertain lead time.

The third stage is a problem at the end of the cycle, where the number of orders received is known. If the amount received is equal to the order amount, this amount will become the initial inventory amount for the next cycle. If the amount received is less than the amount ordered, there are two possible conditions. First, if the type of drug is complete but the quantity is missing, this deficiency may be expected in accordance with the agreement as long as there is no storage or return to the first stage. Second, if the drug type is incomplete, then the back-order is what is done.

$$Y_i < Q_i \rightarrow \begin{cases} i < M; backorder \\ i = M; order\ (Q_i, r_i) \end{cases}$$

This study considers a continuous review of the one-echelon pharmaceutical supply chain, the hospital. The model has been developed for multi-products. The model considers the limitations of storage capacity and CSL to be constraints, while the demand and lead time are assumed to be uncertain and the amount received does not match the order.

MATHEMATICAL FORMULATION

In general, this problem can be formulated as the following stochastic programming model:

Stage I

$$\min TC = TC_1 + E[TC_2] + E[TC_3] \qquad (1)$$

Subject to:

$$I(t) \geq 0; 0 \leq t \leq t_2 \qquad (2)$$

$$v = \begin{cases} 0; & I(t) > r \; product\ not\ ordered \\ 1; & I(t) \leq r\ ;\ product\ ordered \end{cases} \qquad (3)$$

$$n = \left\lceil \frac{Q}{z} \right\rceil \geq 1 \qquad (4)$$

$$0 \leq TC \leq C \qquad (5)$$

$$C = C_1 + C_2 \qquad (6)$$

$$0 \leq f \leq W \qquad (7)$$

$$C, C_1, C_2, L, c_b, h > 0 \qquad (8)$$

$$0 \leq \theta \leq 1 \qquad (9)$$

$$0 \leq a \leq 1 \qquad (10)$$

Description:

TC = total cost
TC_1 = total cost in stage 1
$E[TC_2]$= total cost expectation at stage 2
$E[TC_3]$= total cost expectation at stage 3

Eq. (2) guarantees the amount of inventory if t is not negative. Eq. (3) is a variable that decides whether or not to place an order. Eq. (4) ensures that the number of lots ordered is at least 1. Eq. (5) provides that the total costs incurred do not exceed the budget. Eq. (6) explains that the hospital budget is a hospital budget plus a bank loan. Eq. (7) ensures that the storage capacity of the drugs does not exceed the room capacity. Eq. (8) ensures that all budgets and costs are positive. Eq. (9) sets the expiration rate and Eq. (10) sets the interest rate for loans to banks.

This stage occurs at interval [0, t_2] (Figure 1), the decision is to determine the optimal order number (Q_i) and the reorder point (r_i) for product I which minimizes the total inventory cost (TC). Taking into account the uncertainty of demand and the length of time.

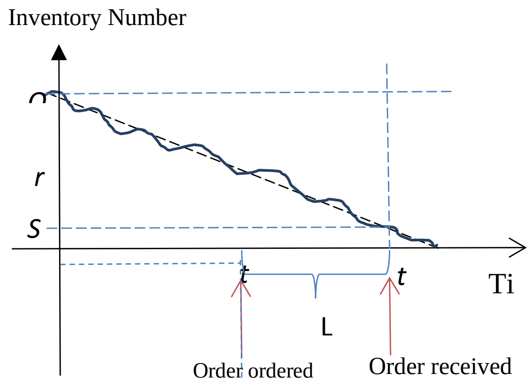

Figure 1. Inventory Model (Q_i, r_i) with Stochastic Demand and lead time

Orders are requested when the inventory amount drops to reorder point (r_i), where this point is determined by taking into account the demand rate during the lead time plus the safety stock, SS in order to avoid stock-out. The reorder point is formulated as follows:

$$r_i = E[D_{Li}(t)] + SS_{\alpha_i} \qquad (11)$$

Description:

r_i = reorder point product *i*
$E[D_{Li}(t)]$= expectation of demand for product *i* during lead time
SS_{α_i}= $1 - \alpha$ = safety stock of the product *i*; with CSL α = The opportunity for the product *i* cannot be fulfilled

The total number of requests during the lead time is shown in Figure 1 as follows:

$$D_{Li}(t) = \int_{t=t_1}^{L} D_i(t)dt$$

$$E[D_i(t)] = E\left[\int_{t=t_1}^{L} D_i(t)dt\right]$$

$$E[D_i(t)] = E[L].\left\{E[D_i(t)]\left[\int_{t=t_1}^{n} D_i(t)dt \middle| L = n\right]\right\}$$

$$E[D_i(t)] = E[L]\left\{\int_{t=t_1}^{n} E[D_i(t)] \middle| D_i(t)|L = n\right\}$$

It is assumed that stochastic demand is normally distributed; $D_{Li}(t) \sim N\left(\mu_{D_{Li}}, \sigma_{D_{Li}}^2\right)$

$$E[D_i(t)] = E[L]\left(\int_{t=t_1}^{L} \mu_{D_i}(t)dt\right)$$

$$E[D_i(t)] = E[L]\left[L.\mu_{D_i}\right]$$

$$E[D_i(t)] = \mu_{D_i}\mu_L$$

The amount of the stock-out shall be specified as

$$\alpha_i = P(D_{Li}(t) > r_i)$$

The following is obtained from Equation (11)

$$\alpha_i = P\left(D_{Li}(t) > E[D_{Li}(t)] + SS_{\alpha_i}\right)$$

$$\alpha_i = P\left(Z > \frac{\left(E[D_{Li}(t)] + SS_{\alpha_i}\right) - E[D_{Li}(t)]}{\sqrt{Var[D_{Li}(t)]}}\right)$$

$$\alpha_i = P\left(Z > \frac{SS_{\alpha_i}}{\sqrt{Var[D_{Li}(t)]}}\right)$$

Let $Z = Z_{\alpha_i}$, then

$$Z_{\alpha_i} = \frac{SS_{\alpha_i}}{\sqrt{Var[D_{Li}(t)]}}$$

$$SS_{\alpha_i} = Z_{\alpha_i}\sqrt{Var[D_{Li}(t)]} \quad (13)$$

Furthermore, from the equation $D_{Li}(t) = \int_{t=t_1}^{L} D_i(t)dt$, the variance is determined as follows:

$$Var[D_{Li}(t)] = Var\left[\int_{t=t_1}^{L} D_i(t)dt\right] = E\left[\left(\int_{t=t_1}^{L} D_i(t)dt\right)^2\right] - \left(E\left[\int_{t=t_1}^{L} D_i(t)dt\right]\right)^2$$

It is assumed that stochastic lead times are normally distributed; $\sim N(\mu_L, \sigma_L^2)$, then:

$$Var[D_{Li}(t)] = E[L].\left[E[D_i(t)]\left[\left(\int_{t=t_1}^{L} D_i(t)dt\right)^2\right]\right] - \left(\mu_{D_i}\mu_L\right)^2$$

$$Var[D_{Li}(t)] = E[L].\left[E[D_i(t)]\left[\left(\int_{t=t_1}^{n} D_i(t)dt\right)^2\middle| L = n\right]\right] - \left(\mu_{D_i}\mu_L\right)^2$$

$$Var[D_{Li}(t)] = E[L]\left[Var[D_i(t)]\left[\left(\int_{t=t_1}^{n} D_i(t)dt\right)^2\middle| L = n\right] + \left(E[D_i(t)]\left[\int_{t=t_1}^{L} D_i(t)dt\middle| L = n\right]\right)^2\right] - \left(\mu_{D_i}\mu_L\right)^2$$

$$Var[D_{Li}(t)] = E[L]\left[Var[D_i(t)]\left[\int_{t=t_1}^{n} D_i(t)dt\middle| L = n\right]\right] + E[L]\left[\left(\int_{t=t_1}^{L} D_i(t)dt\middle| L = n\right)^2\right] - \left(\mu_{D_i}\mu_L\right)^2$$

$$Var[D_{Li}(t)] = E[L]\left[\int_{t=t_1}^{n} [\![Var[D_i(t)]\!]D_i(t)][D_i(t)|L = n] + E[L]\left[\left(\int_{t=t_1}^{n}(D_i(t)|L = n)\right)^2\right]\right] - \left(\mu_{D_i}\mu_L\right)^2$$

$$Var[D_{Li}(t)] = E[L]\left[n.\sigma_{D_i(t)}^2\middle|L = n\right] + E[L]\left[\left(n\mu_{D_i(t)}\middle|L = n\right)^2\right] - \left(\mu_{D_i}\mu_L\right)^2$$

$$Var[D_{Li}(t)] = E[L]\left[L.\sigma_{D_i(t)}^2\right] + E[L]\left[\left(L\mu_{D_i(t)}\right)^2\right] - \left(\mu_{D_i}\mu_L\right)^2$$

$$Var[D_{Li}(t)] = \mu_L\sigma_{D_i(t)}^2 + \mu_{D_i(t)}^2E[L].L^2 - \left(\mu_{D_i}\mu_L\right)^2$$

$$Var[D_{Li}(t)] = \mu_L\sigma_{D_i(t)}^2 + \mu_{D_i(t)}^2(\sigma_L^2 + \mu_L^2) - \left(\mu_{D_i}\mu_L\right)^2$$

$$Var[D_{Li}(t)] = \mu_L\sigma_{D_i(t)}^2 + \mu_{D_i(t)}^2\sigma_L^2 + \mu_L^2\mu_{D_i(t)}^2 - \mu_L^2\mu_{D_i(t)}^2$$

$$Var[D_{Li}(t)] = \mu_L\sigma_{D_i(t)}^2 + \mu_{D_i(t)}^2\sigma_L^2$$

From equation (11)-(14), the reorder point of product i is obtained as:

$$r_i = E[D_{Li}(t)] + SS_{\alpha_i}$$

$$r_i = \mu_{D_i}\mu_L + Z_{\alpha_i}\sqrt{Var[D_{Li}(t)]}$$

$$r_i = \mu_{D_i(t)}\mu_L + Z_{\alpha_i}\sqrt{\mu_L\sigma_{D_i(t)}^2 + \mu_{D_i(t)}^2\sigma_L^2} \quad (15)$$

Whereas the order amount is determined as:

$$Q_i = \mu_{D_i}.T \quad (16)$$

The purchase price for a number of Q_i is

$$c_{bi}.v_i.Q_i = c_{bi}.v_i.\mu_{D_i}.T \quad (17)$$

Description:

v_i = decision variable for purchasing product i,

$$v_i = \begin{cases} 0; I_i(t) > r_i\ product\ i\ not\ ordered \\ 1; I_i(t)\ \leq r_i;\ product\ i\ ordered \end{cases}$$

$$\text{Ordering cost} = c_{oi}.v_i\left[\frac{Q_i}{z_i}\right] = c_{oi}.v_i.n_i \quad (18)$$

Description:

z_i = maximum number of packs per lot

n_i = number of lots for each product i

In addition, products that are not sold during this period will incur storage charges. The number of product i that are not sold is formulated as follows:

$$\int_0^{t_2}\{I_i(t) - E[D_i(t)]\}dt = \int_0^{t_2}(I_i(t) - \mu_{D_i(t)})dt$$

The storage costs for product i per unit time are therefore as follows:

$$h_i\left[\int_0^{t_2}(I_i(t) - \mu_{D_i(t)})dt\right] \quad (19)$$

In addition, if it is assumed that the capital held by the hospital is the accumulation of hospital funds and loan funds, in this case:

$$C = C_1 + C_2$$

Description:

C_1 = hospital funds themselves

C_2 = loan funds to the bank, with a loan interest of a per unit time

The interest expense will then be: aC_2. If this interest expense is charged for all items in the hospital inventory and the selling price of each drug has taken into account the interest and profit margins, then the product that is still subject to the interest expense in the inventory is the unsold product. Interest expense is therefore determined as:

$$\frac{aC_2}{\int_0^{t_2} I_i(t)dt}\left\{\int_0^{t_2} I_i(t) - \mu_{D_i(t)}dt\right\}$$

And from eq. (9), the total cost of the unsold product is as follows:

$$\left[h_i + \frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\left\{\int_0^{t_2} I_i(t) - \mu_{D_i(t)}dt\right\} \quad (20)$$

Products which have expired during this period include:

$$\int_0^{t_2}\theta_i I_i(t)dt \quad (14)$$

Consequently, the costs due to expired products are as follows:

$$\left[c_{bi} + \frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\int_0^{t_2}\theta_i I_i(t)dt \quad (21)$$

Thus, for stage I, from equation (17) to (21), the total cost of inventory of the product i is formulated as:

$$TC_{1i} = purcasing\ cost + ordering\ cost + holding\ cost + expiration\ fee$$

$$TC_{1i} = c_{bi}.v_i.\mu_{D_i}.T + c_{bi}.v_i.n_i + \left[h_i + \frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\left\{\int_0^{t_2} I_i(t) - \mu_{D_i(t)}dt\right\} + \left[c_{bi} + \frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\int_0^{t_2}\theta_i I_i(t)dt \quad (22)$$

And the total cost of inventory for all products is as follows:

$$TC_{1i} = \sum_{i=1}^{M}\left\{c_{bi}.v_i.\mu_{D_i}.T + c_{bi}.v_i.n_i + \left[h_i + \frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\left\{\int_0^{t_2} I_i(t) - \mu_{D_i(t)}dt\right\} + \left[c_{bi} + \frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\int_0^{t_2}\theta_i I_i(t)dt\right\} \quad (23)$$

Thus, the objective function is:

$$\min TC_1 = \min \sum_{i=1}^{M}\left\{c_{bi}.v_i.\mu_{D_i}.T + c_{oi}.v_i.n_i + \left[h_i + \frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\left\{\int_0^{t_2} I_i(t) - \mu_{D_i(t)}dt\right\} + \left[c_{bi} + \frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\int_0^{t_2}\theta_i I_i(t)dt\right\} \quad (24)$$

Subject to:

$$I_i(t) \geq 0; 0 \leq t \leq t_2; i = 1,2,\dots,M \quad (25)$$

$$v_i = \begin{cases} 0; I_i(t) > r_i \; product\ i\ not\ ordered \\ 1; I_i(t) \leq r_i; \; product\ i\ ordered \end{cases} \quad (26)$$

$$n_i = \left\lceil\frac{Q_i}{z_i}\right\rceil \geq 1 \quad (27)$$

$$0 \leq TC \leq C \quad (28)$$

$$C = C_1 + C_2 \quad (29)$$

$$0 \leq \sum_{i=1}^{M} f_i \leq W \quad (30)$$

$$C, C_1, C_2, L, c_{bi}, h_i > 0 \quad (31)$$

$$0 \leq \theta_i \leq 1 \quad (32)$$

$$0 \leq a \leq 1 \quad (33)$$

Description:

TC = total cost

TC_1 = total cost in stage 1;

$E[TC_2]$ = total cost expectation at stage 2;

$E[TC_3]$ = total cost expectation at stage 3.

Eq. (25) guarantees the amount of inventory if t is not negative. Eq. (26) is a variable that decides whether or not to place an order. Eq. (27) ensures that the number of lots ordered is at least 1. Eq. (28) provides that the total costs incurred do not exceed the budget. Eq. (29) explains that the hospital budget is a hospital budget plus a bank loan. Eq. (30) ensures that the storage capacity of the drug does not exceed the capacity of the room. Eq. (31) ensures that all budgets and costs are positive. Eq. (32) sets the expiration rate and Eq. (33) sets the interest rate on loans to banks.

Stage II

This stage can be formulated as a stochastic programming model as follows:

$$\min \xi_1 = TC_2 + E[TC_3] \quad (35)$$

Subject to:

$$\sum_{i=1}^{M} I_{si}(t) \leq S_{imaks} \quad (36)$$

$$0 \leq TC_s \leq C \quad (37)$$

$$I_{si}(t) < 0; t_2 \leq t \leq t_3; i = 1,2,\dots,M \quad (38)$$

$$v_i = \begin{cases} 0; I_i(t) > r_i \; product\ i\ not\ ordered \\ 1; I_i(t) \leq r_i; \; product\ i\ ordered \end{cases} \quad (39)$$

$$n_i = \left\lceil\frac{Q_i}{z_i}\right\rceil \geq 1 \quad (40)$$

$$0 \leq TC \leq C \quad (41)$$

$$0 \leq \sum_{i=1}^{M} f_i \leq W \quad (42)$$

$$c_{si} > c_{bi} > 0 \quad (43)$$

$$C, L, c_{si}, h_i > 0 \quad (44)$$

$$0 \leq \alpha_i \leq 1 \quad (45)$$

The $[t_1, t_3]$ interval (Figure 1) is the lead time period. At this stage, if demand during lead time (X_i) exceeds expectations, where $X_i > r_i$ is expected, there will be a stock-out. This stock-out will be overturned with a complete backordered (not considering a loss sale). The challenge at this stage is uncertainty about lead time and demand.

In addition, if there is a stock-out at interval $[t_2, t_3]$, in this case $X_i > r_i$, then a backorder will be made for all products i experiencing stock-out. The maximum amount of stock-out allowed depends on the hospital level service (CSL) policy, where $CSL = 1 - \alpha$.

There are two backorder scenarios at this stage, namely:

1. Products are received on the same day.
   That is, the purchase of the drug i-th that had been stored on the same day was received.
   The purchase price of the product with this backorder is specified as $c_{si} > c_{bi}$.
   The number of products i order is as follows:

$$I_{si}(t) = -\int_{t=t_2}^{t_3} X_i(t) - r_i dt \quad (46)$$

Purchase price of the whole product i backorder:

$$c_{si}v_i\left[-\int_{t=t_2}^{t_3} X_i(t) - r_i dt\right] \quad (47)$$

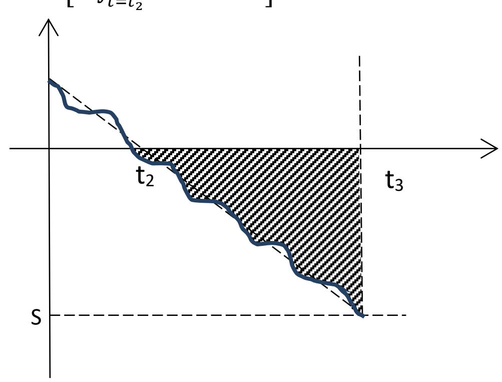

Figure 2: Inventory Model with Stochastic Demand and Lead Time When Stock Out

The total cost of the backorder for the product i is therefore:

$$TC_{si} = backorder\ purchase\ cost + shipping\ cost$$

$$TC_{si} = c_{si}v_i\left[-\int_{t=t_2}^{t_3} X_i(t) - r_i dt\right] + c_{oi}.v_i.n_i \quad (49)$$

And the total cost of all products during the lead time when stock-out is as follows:

$$TC_s = \sum_{i=1}^{M}\left[c_{si}v_i\left[-\int_{t=t_2}^{t_3} X_i(t) - r_i dt\right] + c_{oi}.v_i.n_i\right] \quad (50)$$

Next, the objective function is defined as:

$$min\ TC_s = \min\left\{\sum_{i=1}^{M}\left[c_{si}v_i\left[-\int_{t=t_2}^{t_3} X_i(t) - r_i dt\right] + c_{oi}.v_i.n_i\right]\right\} \quad (51)$$

Subject to:

$$\sum_{i=1}^{M} I_{si}(t) \leq S_{imaks} \quad (52)$$

$$0 \leq TC_s \leq C \quad (53)$$

$$I_{si}(t) < 0; t_2 \leq t \leq t_3; i = 1,2,\dots,M \quad (54)$$

$$v_i = \begin{cases} 0; I_i(t) > r_i\ product\ i\ not\ ordered \\ 1; I_i(t)\ \leq r_i;\ product\ i\ ordered \end{cases} \quad (55)$$

$$n_i = \left|\frac{Q_i}{z_i}\right| \geq 1 \quad (56)$$

$$0 \leq TC \leq C \quad (57)$$

$$0 \leq \sum_{i=1}^{M} f_i \leq W \quad (58)$$

$$c_{si} > c_{bi} > 0 \quad (59)$$

$$C, L, c_{si}, h_i > 0 \quad (60)$$

2. Products are received at the same time as the previous order (Order Q_i)

That is, this backorder is experiencing uncertainty in terms of lead time. As a result, the number of demand during the lead time is also uncertain.

Assuming demand during lead time X_i is stochastic, which is normally distributed; $X_i{\sim}N(\mu_{X_i}, \sigma_{X_i}^2)$.

Suppose the hospital service level is as: $1-\alpha$.

Then, the opportunity for a stock-out for the product i is:

$$\alpha_i = P(X_i > r_i) \quad (61)$$

so, the backorder number is determined as:

$$I_{si}(t) = (expectation\ demand\ in\ lead\ time).P(X_i > r_i)$$

$$I_{si}(t) = E[X_i].\alpha_i \quad (62)$$

Next,

$$E[X_i] = E\left[\int_{t_2}^{L} X_i(t)dt\right]$$

$$E[X_i] = E[L].E[X_i(t)]\left\{\int_{t_2}^{m} X_i(t)dt \middle| L = m\right\}$$

$$E[X_i] = E[L].\left[\int_{t_2}^{m} E[X_i(t)]dt \middle| X_i(t)|L = m\right]$$

$$E[X_i] = E[L].\left[\int_{t_2}^{L} \mu_{X_i(t)}dt\right]$$

$$E[X_i] = E[L].\left[L.\mu_{X_i}\right]$$

$$E[X_i] = \mu_{X_i}.\mu_L \quad (63)$$

From Equations (62) and (63) obtained

$$I_{si}(t) = \mu_{X_i}\mu_L.\alpha_i \quad (64)$$

The purchase price of the product i backorder is:

$$c_{si}v_i(\mu_{X_i}\mu_L.\alpha_i) \quad (65)$$

The shipping cost for product i is:

$$c_{0i}.v_i.\left|\frac{Q_i}{z_i}\right| = c_{oi}.v_i.n_i \quad (66)$$

The total backorder fee for the product i is:

$$TC_{2i} = c_{si}v_i\left(\mu_{X_i}\mu_L.\alpha_i\right) + c_{oi}.v_i.n_i \quad (67)$$

Therefore, the total backorder cost for all products is:

$$TC_2 = \sum_{i=1}^{M}\left\{c_{si}v_i\left(\mu_{X_i}\mu_L.\alpha_i\right) + c_{oi}.v_i.n_i\right\} \quad (68)$$

The objective function is defined as:

$$\min \xi_1 = \min\left[\sum_{i=1}^{M}\left\{c_{si}v_i\left(\mu_{X_i}\mu_L.\alpha_i\right) + c_{oi}.v_i.n_i\right\}\right] \quad (69)$$

Subject to:

$$\sum_{i=1}^{M} I_{si}(t) \leq S_{imaks} \quad (70)$$

$$0 \leq TC_s \leq C \quad (71)$$

$$I_{si}(t) < 0; t_2 \leq t \leq t_3; i = 1,2,\dots,M \quad (72)$$

$$v_i = \begin{cases} 0; I_i(t) > r_i\ product\ i\ not\ ordered \\ 1; I_i(t)\ \leq r_i;\ product\ i\ ordered \end{cases} \quad (73)$$

$$n_i = \left|\frac{Q_i}{z_i}\right| \geq 1 \quad (74)$$

$$0 \leq TC \leq C \quad (75)$$

$$0 \leq \sum_{i=1}^{M} f_i \leq W \quad (76)$$

$$c_{si} > c_{bi} > 0 \quad (77)$$

$$C, L, c_{si}, h_i > 0 \quad (78)$$

$$0 \leq \alpha_i \leq 1 \quad (79)$$

Description:

Eq. (70) requires that the number of stock-outs does not exceed the maximum number of stock-outs allowed under the Hospital CSL policy. Eq. (71) ensures that the total cost of stock-outs does not exceed the budget. Eq. (72) shows that the stock is calculated in $[t_2, t_3]$ intervals. Eq. (73) is a variable that decides whether or not to place an order. Eq. (74) shows the minimum number of batches 1. Eq. (75) shows that the total cost of inventories does not exceed the budget. Eq. (76) shows that the total capacity does not exceed the room capacity. Eq. (77) shows a penalty fee due to a stock out exceeding the normal purchase costs. Eq. (78) shows that the budget, lead time, penalty fees and storage costs are positive. Eq. (79) shows the probability of a stock-out occurring between 0 and 1.

Stage III

This stage can be formulated as:

$$\min \xi_2 = E[TC_3] \tag{80}$$

Subject to:

$$I_i(t) \geq 0; 0 \leq t \leq t_2; i = 1,2,\dots,M \tag{81}$$

$$v_i = \begin{cases} 0; I_i(t) > r_i \; product\ i\ not\ ordered \\ 1; I_i(t) \leq r_i; \; product\ i\ ordered \end{cases} \tag{82}$$

$$n_i = \left\lceil \frac{Y_i}{z_i} \right\rceil \geq 1 \tag{83}$$

$$0 \leq TC \leq C \tag{84}$$

$$C = C_1 + C_2 \tag{85}$$

$$0 \leq \sum_{i=1}^{M} f_i \leq W \tag{86}$$

$$C, C_1, C_2, L, c_{bi}, h_i > 0 \tag{87}$$

$$0 \leq \theta_i \leq 1 \tag{88}$$

$$0 \leq \alpha_i \leq 1 \tag{89}$$

$$0 \leq a \leq 1 \tag{90}$$

Description:

$E[TC_3]$ = Total cost expectations in stage III

Eq. (81) shows that the amount of inventory is positive. Eq. (82) is a variable that decides whether or not to place an order. Eq. (83) shows the number of batches ordered at least 1. Eq. (84) shows that the total cost does not exceed the budget. Eq. (85) shows that the budget consists of hospital and bank loans. Eq. (86) shows that the capacity of the product does not exceed the capacity of the room. Eq. (87) shows that the budget, lead time, purchase and storage costs are positive. Eqs. (88)-(90) shows the expiration rate, stock-out and bank interest rates between 0 and 1.

This stage is the stage at which the realization of the demand, the lead time and the number of orders received are known, i.e. $t = t_3$. If the amount received by Y_i matches the order $Y_i = Q_i$ this will be the initial inventory amount for the next cycle.

If the amount received (Y_i) is smaller than the order, i.e. $Y_i < Q_i$, where:

$$Y_i < Q_i \rightarrow \begin{cases} i < M; \text{backorder} \\ i = M; order\ (Q_i, r_i) \end{cases} \tag{91}$$

It is assumed that the amount received depends on the amount ordered (Uthayakumar & Priyan, 2013), written as:

$$E[Y_i|Q_i] = \tau_i Q_i \tag{92}$$

Description:

τ_i = bias factor amount received against the ordered amount; $0 \leq \tau_i \leq 1$

Where the variance of the number of products i received is determined as:

$$var(Y_i) = \sigma_{i1}^2 + \sigma_{i2}^2 Q_i^2 \tag{93}$$

Description:

$\sigma_{i1}^2, \sigma_{i2}^2 > 0;\ \sigma_{i1}^2 \gg \sigma_{i2}^2$

For product i, if $\sigma_{i2}^2 = 0$, then the standard deviation of the amount received does not depend on the amount ordered, and if $\sigma_{i1}^2 = 0$, then the standard deviation of the amount received is proportional to the amount ordered. Thus from Equation (73), obtained:

$$TC_3 = \sum_{i=1}^{M} \left\{ c_{bi} v_i . Y_i + c_{bi} . v_i . \left\lceil \frac{Y_i}{z_i} \right\rceil + \left[h_i + \frac{aC_2}{\int_0^{t_2} I_i(t)dt} \right] \int_0^{t_2} \left\{ I_i(t) - \frac{\int_0^{t_2} D_i(t)dt}{T} \right\} dt + \left[c_{bi} + \frac{aC_2}{\int_0^{t_2} I_i(t)dt} \right] \int_0^{t_2} \theta_i I_i(t)dt \right\} \tag{94}$$

Next,

$$\frac{E[Y_i|Q_i]}{D_i(t)} = \frac{\tau_i Q_i}{D_i(t)} \tag{95}$$

From Equation (91),

$$E[Y_i] = E[Y_i^2] - [E[Y_i]]^2$$

$$\sigma_{i1}^2 + \sigma_{i2}^2 Q_i^2 = E[Y_i^2] - [E[Y_i]]^2$$

$$\sigma_{i1}^2 + \sigma_{i2}^2 Q_i^2 = E[Y_i^2] - \tau_i^2 Q_i^2$$

$$E[Y_i^2] = \sigma_{i1}^2 + (\sigma_{i2}^2 + \tau_i^2) Q_i^2$$

$$E[Y_i|Q_i] = \sigma_{i1}^2 + (\sigma_{i2}^2 + \tau_i^2) Q_i^2 \tag{96}$$

Thus obtained from Equations (91) and (96):

$$E[TC_{3i}|Q_i] = \sum_{i=1}^{M} \left\{ c_{bi} v_i . Y_i + c_{bi} . v_i . \left\lceil \frac{Y_i}{z_i} \right\rceil + \left[h_i + \frac{aC_2}{\int_0^{t_2} I_i(t)dt} \right] \frac{E[Y_i^2|Q_i]}{\int_0^{t_2} \left\{ I_i(t) - \frac{\int_0^{t_2} D_i(t)dt}{T} \right\} dt} + E[Y_i^2|Q_i] \left[c_{bi} + \frac{aC_2}{\int_0^{t_2} I_i(t)dt} \right] \int_0^{t_2} \theta_i I_i(t)dt \right\}$$

$$E[TC_{3i}|Q_i] = \sum_{i=1}^{M}\left\{c_{bi}v_i.Y_i + c_{bi}.v_i.\left\lceil\frac{Y_i}{z_i}\right\rceil + \left[h_i + \frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\frac{\sigma_{i1}^2+(\sigma_{i2}^2+\tau_i^2)Q_i^2}{\int_0^{t_2}\left\{I_i(t)-\frac{\int_0^{t_2} D_i(t)dt}{T}\right\}dt} + \left(\sigma_{i1}^2 + (\sigma_{i2}^2+\tau_i^2)Q_i^2\right)\left[c_{bi} + \frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\int_0^{t_2}\theta_i I_i(t)dt\right\} \quad (97)$$

From the renewal reward theorem, the expected total cost for product *i* is obtained as:

$$E[TC_{3i}] = \frac{E[TC(Y_i|Q_i)]}{\frac{E[Y_i|Q_i]}{D_i(t)}}$$

So, from Equations (92), (96) and (97), we get:

$$E[TC_{3i}] = \frac{D_i(t)\left(c_{bi}v_i.Y_i+c_{bi}.v_i.\left\lceil\frac{Y_i}{z_i}\right\rceil\right)}{\tau_iQ_i} + \frac{\left[h_i+\frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\left[\sigma_{i1}^2+(\sigma_{i2}^2+\tau_i^2)Q_i^2\right]}{2\tau_iQ_i} + \left(\sigma_{i1}^2 + (\sigma_{i2}^2 + \tau_i^2)Q_i^2\right)\left[c_{bi} + \frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\left\{\int_0^{t_2}\theta_i I_i(t)dt\right\} + \frac{c_{si}.v_i\int_{t_2}^{t_3} -\alpha_i I_i(t)dt}{\tau_iQ_i} \quad (98)$$

Next, the expected total cost for all products is:

$$E[TC_3] = \sum_{i=1}^{M}\left\{\frac{D_i(t)\left(c_{bi}v_i.Y_i+c_{bi}.v_i.\left\lceil\frac{Y_i}{z}\right\rceil\right)}{\tau_iQ_i} + \frac{\left[h_i+\frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\left[\sigma_{i1}^2+(\sigma_{i2}^2+\tau_i^2)Q_i^2\right]}{2\tau_iQ_i} + \left(\sigma_{i1}^2 + (\sigma_{i2}^2 + \tau_i^2)Q_i^2\right)\left[c_{bi} + \frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\left\{\int_0^{t_2}\theta_i I_i(t)dt\right\} + \frac{c_{si}.v_i\int_{t_2}^{t_3} -\alpha_i I_i(t)dt}{\tau_iQ_i}\right\} \quad (99)$$

The objective functions are:

$\min \xi_1 = E[TC_3]$

$min\ E[TC_3] =$

$$\min\left[\sum_{i=1}^{M}\left\{\frac{D_i(t)\left(c_{bi}v_i.Y_i+c_{bi}.v_i.\left\lceil\frac{Y_i}{z_i}\right\rceil\right)}{\tau_iQ_i} + \frac{\left[h_i+\frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\left[\sigma_{i1}^2+(\sigma_{i2}^2+\tau_i^2)Q_i^2\right]}{2\tau_iQ_i} + \left(\sigma_{i1}^2 + (\sigma_{i2}^2 + \tau_i^2)Q_i^2\right)\left[c_{bi} + \frac{aC_2}{\int_0^{t_2} I_i(t)dt}\right]\left\{\int_0^{t_2}\theta_i I_i(t)dt\right\} + \frac{c_{si}.v_i\int_{t_2}^{t_3} -\alpha_i I_i(t)dt}{\tau_iQ_i}\right\}\right] \quad (100)$$

Subject to:

$$I_i(t) \geq 0; 0 \leq t \leq t_2; i = 1,2,\dots,M \quad (101)$$

$$v_i = \begin{cases} 0; I_i(t) > r_i\ product\ i\ not\ ordered \\ 1; I_i(t)\ \leq r_i;\ product\ i\ ordered \end{cases} \quad (102)$$

$$n_i = \left\lceil\frac{Y_i}{z_i}\right\rceil \geq 1 \quad (103)$$

$$0 \leq TC \leq C \quad (104)$$

$$C = C_1 + C_2 \quad (105)$$

$$0 \leq \sum_{i=1}^{M} f_i \leq W \quad (106)$$

$$C, C_1, C_2, L, c_{bi}, h_i > 0 \quad (107)$$

$$0 \leq \theta_i \leq 1 \quad (108)$$

$$0 \leq \alpha_i \leq 1 \quad (109)$$

$$0 \leq a \leq 1 \quad (110)$$

CONCLUSION

Management of pharmaceutical inventory in hospitals has become a critical challenge for health care systems. This paper develops a multi-stage stochastic programming model to optimize the pharmaceutical inventory costs with uncertainty in demand, lead time and received quantity. This model assumes a continuous review policy, with a (Q, r) model for multi-products in one echelon pharmacy supply chain, i.e. a hospital. Stock out is overcome by backorder entirely, without considering loss sale.

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