Capacity Optimization model for Hospital Management Problem under Uncertainty

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ABSTRACT

Capacity management planning involves predicting the quantity and particular attributes of resources required to deliver health care service for hospitals at specified levels of cost and quality. For inpatient hospital care, capacity management requires information on beds and nursing staff capacity. This paper presents a capacity optimization model that gives insight into required nursing staff capacity and opportunities to improve capacity utilization on a ward level with some uncertainty parameters. A capacity model is developed to calculate required nursing staff capacity. The uncertainty turns up on the availability schedule of staff and the number of patient.

INTRODUCTION

Healthcare deliveries planning is essential in order to achieve a balance between the quality of the health care offered and the price of the service provided. Such planning includes the measurement of the amount and specific characteristics of the funds needed to provide health care services at a defined price and quality. Overall, effective healthcare capacity planning must answer a number of issues, including the length of the planning horizon, the level of service conducted, the type of service given, the quantity, capacity, cost and resources available or required (i.e. doctors, nurses, technicians, medical and clinical support personnel, facilities including buildings, rooms, beds, storage areas, laboratory tests and monitoring equipment, or any other component that contributes to the provision of health care) as well as the customer service metrics or performance measures expected for the facility. While capacity planning has confronted health decision-makers and scholars for decades [1,2] and [3], this issue has been addressed with a renewed sense depends on urgency.

This requires information on bed capacity and the capacity of nursing staff for inpatient care facilities in the hospital on a daily and annual basis. Quantitative models may be used to identify capacity needs for various planning purposes and for short, medium and long-term planning matters. Even though a number of valuable models are outlined in global literature [4], [5] and [6], some of them are complicated to implement in reality because they involve a lot of information and clerical work [1].

Stochastic Programs: General Formulation

We define the stochastic (linear) program as the following model

\[
\begin{align*}
\min \quad & \mathbf{g}_0(x, \xi) \\
\text{s.t.} \quad & \mathbf{g}_i(x, \xi) \leq 0, i = 1, \ldots, m, \\
& x \in X \subset \mathbb{R}^n,
\end{align*}
\]

(1)

where \( \xi \) is a random vector varying over a set \( \Xi \subset \mathbb{R}^k \). More precisely, we assume throughout that a family F of
“events”, i.e. subsets of Ξ, and the probability distribution P on F are given. Hence for every subset \( A \subseteq \Xi \) that is an event, i.e. \( A \in F \), the probability \( P(A) \) is known. Furthermore, we assume that the functions \( g_i(x, \cdot) : \Xi \to \Delta \) \( \forall x, i \) are random variables themselves and that the probability distribution P is independent of x.

However, problem (1) is not well defined since the meanings of “min”; as well as of the constraints are not clear at all, if we think of taking a decision on x before knowing the realization of \( G \). Therefore a revision of the modeling process is necessary, leading to so-called deterministic equivalents for (1), which can be introduced in various ways.

**DETERMINISTIC MODEL**

First we formulate a deterministic model for the hospital capacity management problem.

**Notations to be used**

- \( DA_{ij} \): Initial number of doctors with type j in department i
- \( SA_{ij} \): Initial number of nurses with type j in department i
- \( SBA_{ij} \): Initial number of nurse aids type j in department i
- \( D_{ij} \): Type j doctors added to department i
- \( S_{ij} \): Type j nurses added to department i
- \( TPA_{ij} \): Type j nurses with type j aids in department i
- \( TPA_{ij} \): Initial number of common beds in department i
- \( TPU_{ij} \): Initial number of private beds added to department i
- \( TBP_{ij} \): Number of common beds added to department i
- \( bd_{ij} \): The cost of j type doctors in department i
- \( bs_{ij} \): The cost of j type nurses in department i
- \( bsa_{ij} \): The cost of j type nurse aids in department i
- \( bt \): The cost of operating private beds in department i
- \( bo \): The cost of operating common beds in department i
- \( bw_i \): Waiting cost for private beds in department i
- \( bw_u \): Waiting cost for common beds in department i
- \( \alpha_j \): Percentage of type j doctors needed to department i
- \( \beta_j \): Percentage of type j nurses needed to department i
- \( \gamma_j \): Percentage of type j nurse aids in department i
- \( \delta \): Percentage of private beds used as common beds in department i
- \( \mu \): Maximum number of beds that can be added to department i
- \( \rho \): Maximum fund available for department i

**Objective Function**
The objective of this problem is to minimize total operating cost which can be expressed as

\[
\text{Minimize} \sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_j (TP_i + TPU_i) + \sum_{i=1}^{n} \beta_j (TP_i + TPU_i) + \sum_{i=1}^{n} \gamma_j (TP_i + TPU_i) + \sum_{i=1}^{n} \delta_i (TBP_{ij} + TPU_{ij}) + \sum_{i=1}^{n} \mu_i (bw_i(TPU_i + TPU_i))
\]

**Constraints:**

\[\alpha_j (TP_i + TPU_i) \leq D_{ij}; i = 1, 2, \ldots, n; j = 1, 2, \ldots, m\]  

\[\beta_j (TP_i + TPU_i) \leq S_{ij}; i = 1, 2, \ldots, n; j = 1, 2, \ldots, m\]  

\[\gamma_j (TP_i + TPU_i) \leq SBA_{ij}; i = 1, 2, \ldots, n; j = 1, 2, \ldots, m\]  

\[\delta_i (TP_i + TPU_i) \leq TBP_{ij}; i = 1, 2, \ldots, n; j = 1, 2, \ldots, m\]  

\[TP_i + TPU_i \leq \mu; i = 1, 2, \ldots, n\]  

\[\sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_j (TP_i + TPU_i) + \sum_{i=1}^{n} \beta_j (TP_i + TPU_i) + \sum_{i=1}^{n} \gamma_j (TP_i + TPU_i) + \sum_{i=1}^{n} \delta_i (TP_i + TPU_i) + \sum_{i=1}^{n} \mu_i (bw_i(TPU_i + TPU_i)) \geq 0 \text{ and integer}\]  

### EQUIVALENT DETERMINISTIC MODEL

\( P_s \): Probability that scenario \( s \) will \( s \in S \) (set of scenario)

Minimize:

\[
\sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_j (TP_i + TPU_i) + \sum_{i=1}^{n} \beta_j (TP_i + TPU_i) + \sum_{i=1}^{n} \gamma_j (TP_i + TPU_i) + \sum_{i=1}^{n} \delta_i (TP_i + TPU_i) + \sum_{i=1}^{n} \mu_i (bw_i(TPU_i + TPU_i))
\]

**Constraints:**

\[\alpha_j (TP_i + TPU_i) \leq D_{ij}; i = 1, 2, \ldots, n; j = 1, 2, \ldots, m\]  

\[\beta_j (TP_i + TPU_i) \leq S_{ij}; i = 1, 2, \ldots, n; j = 1, 2, \ldots, m\]  

\[\gamma_j (TP_i + TPU_i) \leq SBA_{ij}; i = 1, 2, \ldots, n; j = 1, 2, \ldots, m\]  

\[\delta_i (TP_i + TPU_i) \leq TBP_{ij}; i = 1, 2, \ldots, n; j = 1, 2, \ldots, m\]  

\[TP_i + TPU_i \leq \mu; i = 1, 2, \ldots, n\]  

\[\sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_j (TP_i + TPU_i) + \sum_{i=1}^{n} \beta_j (TP_i + TPU_i) + \sum_{i=1}^{n} \gamma_j (TP_i + TPU_i) + \sum_{i=1}^{n} \delta_i (TP_i + TPU_i) + \sum_{i=1}^{n} \mu_i (bw_i(TPU_i + TPU_i)) \geq 0 \text{ and integer}\]
\[
\gamma_i (TP_i + TU_i) \leq SB_j; i = 1, \ldots, n; j = 1, \ldots, m \quad (16)
\]

\[
\delta_i (TU_i + TUA_i) \leq TBP_i; i = 1, \ldots, n; j = 1, \ldots, m \quad (17)
\]

\[
TP_i + TU_i \leq \mu \quad (18)
\]

\[
\sum_{i=1}^{n} \sum_{j=1}^{m} b_{ij} D_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{m} b_{ij} S_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{m} b_{ij} S_{ij} + \sum_{i=1}^{n} b_{ij} TP_i + \sum_{i=1}^{n} b_{ij} TU_i \leq P_i; i = 1, \ldots, n; s \in S \quad (19)
\]

\[
DA_{ij}, S_{ij}, SB_{ij}, TP_i, TU_i \geq 0 \quad \text{and integer} \quad (20)
\]

**FEASIBLE NEIGHBOURHOOD SEARCH**

Step 1. Solve the relaxed problem. To get the optimal continuous solution.

Step 2. Find the row index in which occur integer infeasibility from the continuous solution.

Step 3. Do a pricing operation as in simplex method for linear programming problem.

Step 4. Calculate vector \( \sigma_{ij} = v_j^T a_j \)

With \( j \) can be found from \( \min_j \left\{ \frac{d_j}{\sigma_{ij}} \right\} \).

If none, take the other non-integer nonbasic or superbasic \( j \).

Step 5. Find the step size \( \alpha_{j^*} = B^{-1} \alpha_{j^*} \).

i.e. solve \( B \alpha_{j^*} = \alpha_{j^*} \) for \( \alpha_{j^*} \).

Step 6. Do a ratio test for maintaining the exchange basis.


**CONCLUSIONS**

This paper provides a model of capacity management for nursing staff under uncertainty. The outcome model would be a massive-scale issue, depending on the number of scenarios. To solve the integer model, we suggest a direct search approach.

**REFERENCES**


