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Fuzzy Model Optimization Using of Giving the Amplitude Scale Factor

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Abstract

Classical time series modeling is often used by practitioners for prediction purposes. Many researchers were already developed a way to predict time series using new methods, one of which is a method of fuzzy method. Fuzzy models were constructed based on the data, commonly not yet have an optimal value of Mean Absolute Error (MAE), which mean MAE on the model can still be reduced. This paper describes how the reduction of MAE on fuzzy model using of giving the amplitude scale factor. Impairment of MAE mathematically demonstrated in the evidence.

Keywords: word; fuzzy, mean absolute error, scale factor, time series

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1. Introduction

Modeling of time series is widely used by practitioners for prediction purposes such as drink production, stock prices, the use of gas, electricity, etc. In addition, classic modeling using ARIMA. The new method has been developed by several researchers. Newer methods, in general, can provide an empirical results are more accurate than previous methods. This paper uses fuzzy method for prediction purposes in the time series. Fuzzy model is optimized using of giving the amplitude scale factor.

Fuzzy theory initiated by Zadeh discovery (1965). Since his discovery, the fuzzy rule-based models is widely applied in various statistical models. Fuzzy inference systems developed by Zadeh (1). Takagi, Sugeno and Kang (TSK) presented a low-complexity, computationally effective models without fuzzy sets within the rule consequences (2,3). The use of fuzzy model for prediction of time series preceded by a (4).

At this time, fuzzy modeling has been frequently used by some researchers. Many fuzzy models that have been developed to improve accuracy. But in general, fuzzy model of development is not done inexact ways, because of the nature of the fuzzy model is to translate linguistic variables into numeric variables. Therefore, (5) attempts to minimize the mean square error (MSE) using translational models. Translation as far as the mean error, proved to be minimize the MSE. (6) also tried to minimize the Mean Absolute Error (MAE) using a similar way. Translation as far as the median error, can reduce MAE.

According to (7), (8) and(9), in addition of the translation, a model is likely to be given scale factor to minimize error, in this case MAE is minimized. This paper describes how to giving scale factor to minimize the RMSE.

2. The scale factor in the model

Given the time series model $\hat{y}_t = f(y_{t-1}, y_{t-2}, \ldots, y_{t-p})$. The model will be modified to minimize the MAE. Modifications done by giving the amplitude with the center of the scale is the line y = c for some constant c.

To simplify the analysis, the actual data and the predicted results of the model to be transformed into $z_i = y_i - c$, $\hat{z}_i = \hat{y}_i - c$ and the center line scale is transformed into the line z = 0. Henceforth, the discussion will be considered using the points that have been transformed as far as –c

Definition 1

MAE in the time series

Given a set of time series values $y_1, y_2, \ldots y_n$ and the prediction value is $\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_n$ with $\hat{y}_1 = y_1 + e_1, \hat{y}_2 = y_2 + e_2, \ldots, \hat{y}_n = y_n + e_n$ Then the *Mean Absolute Error* is calculated using the formula MAE $= \frac{1}{n}\sum_{i=1}^{n} |e_i|$

Given a set of time series values $z_1, z_2, \ldots z_n$, and suppose that the prediction value is $\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_n$, The position of the points a prediction of actual points of time series data, viewed from the line z = 0, will be grouped into four types namely *cross-over, outer, inner* and *precise* as shown in Figure 1, and is defined as follows:

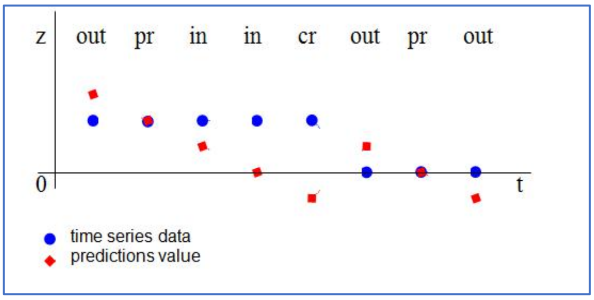

Figure 1. Representation of the point *outer, inner, precise* and *crossover.*

Definition 2

Given point z_i , i=1, 2, ..., n and the line z = 0.

a. If $z_i\hat{z}_i < 0$ then z_i and $\hat{z}_i$ said to be crossover and be written z_i^{cr} and $\hat{z}_i^{cr}$.

b. If $z_i\hat{z}_i \geq 0$ then z_i and $\hat{z}_i$ said to be unilateral.

Definition 3

Given point z_i, i=1, 2, ..., n , the line z = 0 and $\hat{z}_i$ is unilateral

a. If $|\hat{z}_i|>|z_i|$, then the point z_i and $\hat{z}_i$ is called outer, written as z_i^{out} and $\hat{z}_i^{out}$.

b. If $|\hat{z}_i|<|z_i|$, then the point z_i and $\hat{z}_i$ is called inner, written as z_i^{in} and $\hat{z}_i^{in}$.

c. If $\hat{z}_i = z_i$, then the point z_i and $\hat{z}_i$ is called precise, written as z_i^{pr} and $\hat{z}_i^{pr}$.

Values $z_1, z_2, \ldots, z_n$ can be partitioned based on its position relative to the line z = 0, ie, *outer, inner, precise* and *crossover*

Outer: z_h^{out}; *h*= 1, 2, ..., p ,

Inner: z_j^{in}; *j*= 1, 2, ..., q ,

Precise: z_k^{pr}; *k*= 1, 2, ..., r ,

Crossover: z_l^{cr}; *l*= 1, 2, ..., s .

Based on that partition, it is clear that n = p + q + r + s

Giving scale factor means it will have an impact compression or stretching on the model. Stretching or compression will be used to reduce error. The model that are stretched will be compressed and the model that are compressed will be stretched

Definition 4

Given Z the result of the transformation of the realization of a time series, $\widehat{Z}$ is the prediction model with outer z_h^{out} and $\hat{z}_h^{out}$, inner z_j^{in} and $\hat{z}_j^{in}$, precise z_k^{pr} and $\hat{z}_k^{pr}$, as well as crossover z_l^{cr} and $\hat{z}_l^{cr}$, A model $\widehat{Z}$ is said stretched if

$$\sum_{h=1}^{p}\left|\hat{z}_h^{out}\right| + \sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right| > \sum_{j=1}^{q}\left|\hat{z}_j^{in}\right| + \sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|$$

A model that is included in the stretched category need to be compressed to improve accuracy. If the compression is done, then the absolute price number of errors in the points *outer* $\hat{z}_h^{out}$ and *crossover* $\hat{z}_l^{cr}$ will be reduced, but the absolute price number of errors in the points *inner* $\hat{z}_j^{in}$ and *precise* $\hat{z}_k^{pr}$ that is not on the center line of scale, will have increased. Thus, a model can be compressed to minimize error if the absolute price number of errors in the points *outer* $\hat{z}_h^{out}$ and *crossover* $\hat{z}_l^{cr}$ is greater than the absolute price number of errors in the points *inner* $\hat{z}_j^{in}$ and *precise* $\hat{z}_k^{pr}$.

The idea to compress the model is to give a scale factor that can squeeze point $\hat{z}_h^{out}$ and point z_h^{out} which has the shortest distance, this is conducted to ensure that the error of the entire point *outer* $\hat{y}_h^{out}$ will shrink. Selecting a point $\hat{z}_h^{out}$ and point z_h^{out} which has the closest distance, then choose $\frac{z_h^{out}}{\hat{z}_h^{out}}$ the greatest, because the points $\hat{z}_h^{out}$ always further away from z_h^{out} toward the line z = 0.

Theorem 1

Given Z the result of the transformation of the realization of a time series, $\widehat{Z}$ is the prediction model with outer z_h^{out} and $\hat{z}_h^{out}$, inner z_j^{in} and $\hat{z}_j^{in}$, precise z_k^{pr} and $\hat{z}_k^{pr}$, as well as crossover z_l^{cr} and $\hat{z}_l^{cr}$,

If $\sum_{h=1}^{p}\left|\hat{z}_h^{out}\right| + \sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right| > \sum_{j=1}^{q}\left|\hat{z}_j^{in}\right| + \sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|$ and $\alpha = max(\alpha_h)$ with $\alpha_h = \frac{z_h^{out}}{\hat{z}_h^{out}}$ then $\widehat{Z}^* = \alpha\widehat{Z}$ provide $MAE(\widehat{Z}^*) < MAE(\widehat{Z})$.

Proof:

Made $\alpha_h = \frac{z_h^{out}}{\hat{z}_h^{out}}$. Because for every *h* applicable $|\hat{z}_h^{out}| > |z_h^{out}|$ then

$0 < \alpha_h < 1$ (1)

Taken $\alpha = max(\alpha_h : h = 1,2 \ldots, p)$, then for every *h* = 1,2,…,p applicable

$\left|\alpha\hat{z}_h^{out}\right| \geq \left|\alpha_h\hat{z}_h^{out}\right| = \left|z_h^{out}\right|$ (2)

Given

$\hat{z}_h^{out*} = \alpha\hat{z}_h^{out}$, $e_h^{out} = \hat{z}_h^{out} - z_h^{out}$, $e_h^{out*} = \hat{z}_h^{out*} - z_h^{out}$

$\hat{z}_j^{in*} = \alpha\hat{z}_j^{in}$, $e_j^{in} = \hat{z}_j^{in} - z_j^{in}$, $e_j^{in*} = \hat{z}_j^{in*} - z_j^{in}$,

$\hat{z}_k^{pr*} = \alpha\hat{z}_k^{pr}$, $e_k^{pr} = \hat{z}_k^{pr} - z_k^{pr}$, $e_k^{pr*} = \hat{z}_k^{pr*} - z_k^{pr}$,

$\hat{z}_l^{cr*} = \alpha\hat{z}_l^{cr}$, $e_l^{cr} = \hat{z}_l^{cr} - z_l^{cr}$, $e_l^{cr*} = \hat{z}_l^{cr*} - z_l^{cr}$,

then

$$\sum_{h=1}^{p}\left|e_h^{out*}\right| = \sum_{h=1}^{p}\left|\hat{z}_h^{out*} - z_h^{out}\right| = \sum_{h=1}^{p}\left|\alpha\hat{z}_h^{out} - z_h^{out}\right| \quad (3)$$

$$\sum_{j=1}^{q}\left|e_j^{in*}\right| = \sum_{j=1}^{q}\left|\hat{z}_j^{in*} - z_j^{in}\right| = \sum_{j=1}^{q}\left|\alpha\hat{z}_j^{in} - z_j^{in}\right| \quad (4)$$

$$\sum_{k=1}^{r}\left|e_k^{pr*}\right| = \sum_{k=1}^{r}\left|\hat{z}_k^{pr} - z_k^{pr}\right| = \sum_{k=1}^{r}\left|\alpha\hat{z}_k^{pr} - z_k^{pr}\right| \quad (5)$$

$$\sum_{l=1}^{s}\left|e_l^{cr*}\right| = \sum_{l=1}^{s}\left|\hat{z}_l^{cr*} - z_l^{cr}\right| = \sum_{l=1}^{s}\left|\alpha\hat{z}_l^{cr} - z_l^{cr}\right| \quad (6)$$

Given $\sum_{i=1}^{n}\left|e_i^*\right|$ with

$$\sum_{i=1}^{n}\left|e_i^*\right| = \sum_{h=1}^{p}\left|e_h^{out*}\right| + \sum_{j=1}^{q}\left|e_j^{in*}\right| + \sum_{k=1}^{r}\left|e_k^{r*}\right| + \sum_{l}^{s}\left|e_l^{cr*}\right|$$

by looking at the results (3), (4), (5), and (6) then

$$\sum_{i=1}^{n}\left|e_i^*\right| = \sum_{h=1}^{p}\left|\alpha\hat{z}_h^{out} - z_h^{out}\right| + \sum_{j=1}^{q}\left|\alpha\hat{z}_j^{in} - z_j^{in}\right| + \sum_{k=1}^{r}\left|\alpha\hat{z}_k^{pr} - z_k^{pr}\right| + \sum_{l=1}^{s}\left|\alpha\hat{z}_l^{cr} - z_l^{cr}\right|$$

Based on (1), (2), Definition 2, dan Definition 3 then

$$\sum_{i=1}^{n}\left|e_i^*\right| = \sum_{h}^{p}\left(\left|\alpha\hat{z}_h^{out}\right| - \left|z_h^{out}\right|\right) + \sum_{j}^{q}\left(\left|z_j^{in}\right| - \left|\alpha\hat{z}_j^{in}\right|\right) + \sum_{k=1}^{r}\left(\left|z_k^{pr}\right| - \left|\alpha\hat{z}_k^{pr}\right|\right) + \sum_{l=1}^{s}\left(\left|\alpha\hat{z}_l^{cr}\right| + \left|z_l^{cr}\right|\right)$$

$$= \left(\sum_{h}^{p}\left|\alpha\hat{z}_h^{out}\right| - \sum_{h}^{p}\left|z_h^{out}\right|\right) + \left(\sum_{j}^{q}\left|z_j^{in}\right| - \sum_{j}^{q}\left|\alpha\hat{z}_j^{in}\right|\right) + \left(\sum_{k=1}^{r}\left|z_k^{pr}\right| - \sum_{k=1}^{r}\left|\alpha\hat{z}_k^{pr}\right|\right) + \left(\sum_{l=1}^{s}\left|\alpha\hat{z}_l^{cr}\right| + \sum_{l=1}^{s}\left|z_l^{cr}\right|\right) \quad (7)$$

Given $\sum_{i=1}^{n}|e_i|$, $e_h^{out} = \hat{z}_h^{out} - z_h^{out}$, $e_j^{in} = \hat{z}_j^{in} - z_j^{in}$, $e_k^{pr} = \hat{z}_k^{pr} - z_k^{pr}$, $e_l^{cr} = \hat{z}_l^{cr} - z_l^{cr}$.

According to Definition 2, and Definition **3**, then e_i can be partitioned into *outer, inner, precise* and *crosover,* so

$$\sum_{i=1}^{n}|e_i| = \sum_{h=1}^{p}\left|e_h^{out}\right| + \sum_{j=1}^{q}\left|e_j^{in}\right| + \sum_{k=1}^{r}\left|e_k^{pr}\right| + \sum_{l}^{s}\left|e_l^{cr}\right|$$

$$= \sum_{h=1}^{p}\left|\hat{z}_h^{out} - z_h^{out}\right| + \sum_{j=1}^{q}\left|\hat{z}_j^{in} - z_j^{in}\right| + \sum_{k=1}^{r}\left|\hat{z}_k^{pr} - z_k^{pr}\right| + \sum_{l=1}^{s}\left|\hat{z}_l^{cr} - z_l^{cr}\right|$$

Due to the properties on Definition 2 and Definition **3**, then

$$\sum_{i=1}^{n}|e_i| = \sum_{h}^{p}\left(\left|\hat{z}_h^{out}\right| - \left|z_h^{out}\right|\right) + \sum_{j}^{q}\left(\left|z_j^{in}\right| - \left|\hat{z}_j^{in}\right|\right) + \sum_{k=1}^{r}\left(\left|z_k^{pr}\right| - \left|\hat{z}_k^{pr}\right|\right) + \sum_{l=1}^{s}\left(\left|\hat{z}_l^{cr}\right| + \left|z_l^{cr}\right|\right)$$

$$= \left(\sum_{h}^{p}\left|\hat{z}_h^{out}\right| - \sum_{h}^{p}\left|z_h^{out}\right|\right) + \left(\sum_{j}^{q}\left|z_j^{in}\right| - \sum_{j}^{q}\left|\hat{z}_j^{in}\right|\right) + \left(\sum_{k=1}^{r}\left|z_k^{pr}\right| - \sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|\right) + \left(\sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right| + \sum_{l=1}^{s}\left|z_l^{cr}\right|\right) \quad (8)$$

Next will be shown that $MAE(\widehat{Y}^{*}) < MAE(\widehat{Y})$.
According to (7) and (8) then

$\sum_i^n |e_i| - \sum_i^n |e^*_i| = (\sum_h^p |\hat{z}_h^{out}| - \sum_h^p |z_h^{out}|) + (\sum_j^q |z_j^{in}| - \sum_j^q |\hat{z}_j^{in}|)$
$+ (\sum_{k=1}^r |z_k^{pr}| - \sum_{k=1}^r |\hat{z}_k^{pr}|) + (\sum_{l=1}^s |\hat{z}_l^{cr}| + \sum_{l=1}^s |z_l^{cr}|)$
$- (\sum_h^p |\alpha\hat{z}_h^{out}| - \sum_h^p |z_h^{out}|) - (\sum_j^q |z_j^{in}| - \sum_j^q |\alpha\hat{z}_j^{in}|)$
$-(\sum_{k=1}^r |z_k^{pr}| - \sum_{k=1}^r |\alpha\hat{z}_k^{pr}|) - (\sum_{l=1}^s |\alpha\hat{z}_l^{cr}| + \sum_{l=1}^s |z_l^{cr}|)$

According to the commutative properties, then

$\sum_i^n |e_i| - \sum_i^n |e^*_i| = (\sum_h^p |\hat{z}_h^{out}| - \sum_h^p |z_h^{out}|) - (\sum_h^p |\alpha\hat{z}_h^{out}| - \sum_h^p |z_h^{out}|)$
$+(\sum_j^q |z_j^{in}| - \sum_j^q |\hat{z}_j^{in}|) - (\sum_j^q |z_j^{in}| - \sum_j^q |\alpha\hat{z}_j^{in}|)$
$+(\sum_{k=1}^r |z_k^{pr}| - \sum_{k=1}^r |\hat{z}_k^{pr}|) - (\sum_{k=1}^r |y_k^{pr}| - \sum_{k=1}^r |\alpha\hat{y}_k^{pr}|)$
$+(\sum_{l=1}^s |\hat{z}_l^{cr}| + \sum_{l=1}^s |z_l^{cr}|) - (\sum_{l=1}^s |\alpha\hat{z}_l^{cr}| + \sum_{l=1}^s |z_l^{cr}|)$
$=(\sum_{h=1}^p |\hat{z}_h^{out}| - \sum_{h=1}^p |\alpha\hat{z}_h^{out}|) - (\sum_{j=1}^q |\hat{z}_j^{in}| - \sum_{j=1}^q |\alpha\hat{z}_j^{in}|)$
$-(\sum_{k=1}^r |\hat{z}_k^{pr}| - \sum_{k=1}^r |\alpha\hat{z}_k^{pr}|) + (\sum_{l=1}^s |\hat{z}_l^{cr}| - \sum_{l=1}^s |\alpha\hat{z}_l^{cr}|)$

According to the distributive and associative properties, then

$\sum_i^n |e_i| - \sum_i^n |e^*_i| = \sum_{h=1}^p (|\hat{z}_h^{out}| - |\alpha\hat{z}_h^{out}|) - \sum_{j=1}^q (|\hat{z}_j^{in}| - |\alpha\hat{z}_j^{in}|)$
$-\sum_{k=1}^r (|\hat{z}_k^{pr}| - |\alpha\hat{z}_k^{pr}|) + \sum_{l=1}^s (|\hat{z}_l^{cr}| - |\alpha\hat{z}_l^{cr}|)$
$=\sum_{h=1}^p (1-\alpha)|\hat{z}_h^{out}| - \sum_{j=1}^q (1-\alpha)|\hat{z}_j^{in}| - \sum_{k=1}^r (1-\alpha)|\hat{z}_k^{pr}| + \sum_{l=1}^s (1-\alpha)|\hat{z}_l^{cr}|$
$=(1-\alpha)\{\sum_{h=1}^p |\hat{z}_h^{out}| - \sum_{j=1}^q |\hat{z}_j^{in}| - \sum_{k=1}^r |\hat{z}_k^{pr}| + \sum_{l=1}^s |\hat{z}_l^{cr}|\}$
$=(1-\alpha)\{\{\sum_{h=1}^p |\hat{z}_h^{out}| + \sum_{l=1}^s |\hat{z}_l^{cr}|\} - \{\sum_{j=1}^q |\hat{z}_j^{in}| + \sum_{k=1}^r |\hat{z}_k^{pr}|\}\}$ (9)

Because of the terms on Theorem **1** is
$\{\sum_{h=1}^p |\hat{z}_h^{out}| + \sum_{l=1}^s |\hat{z}_l^{cr}|\} > \{\sum_{j=1}^q |\hat{z}_j^{in}| + \sum_{k=1}^r |\hat{z}_k^{pr}|\}$ and $0 < \alpha < 1$, then $\sum_i^n |e_i| - \sum_i^n |e^*_i| > 0$.

Consequently $MAE(\widehat{Z}^{*}) = \frac{1}{n}\sum_i^n |e_i^*| < \frac{1}{n}\sum_i^n |e_i| = MAE(\widehat{Z})$.▓

Theorem **1** does not guarantee that the resulting MAE is the smallest MAE. therefore, when giving scale factor is already done, needs to be double check on the condition of the theorem. If the condition is met, the giving scale factor can be done again

Consequence 2

Given Z the result of the transformation of the realization of a time series, $\widehat{\mathbf{Z}}$ is the prediction model with outer $\mathbf{z_h^{out}}$ and $\hat{\mathbf{z}}_\mathbf{h}^\mathbf{out}$, inner $\mathbf{z_j^{in}}$ and $\hat{\mathbf{z}}_\mathbf{j}^\mathbf{in}$, precise $\mathbf{z_k^{pr}}$ and $\hat{\mathbf{z}}_\mathbf{k}^\mathbf{pr}$ as well as crossover $\mathbf{z_l^{cr}}$ and $\hat{\mathbf{z}}_\mathbf{l}^\mathbf{cr}$,

If $\sum_{\mathbf{h=1}}^{\mathbf{p}} |\hat{\mathbf{z}}_\mathbf{h}^\mathbf{out}| + \sum_{\mathbf{l=1}}^{\mathbf{s}} |\hat{\mathbf{z}}_\mathbf{l}^\mathbf{cr}| = \sum_{\mathbf{j=1}}^{\mathbf{q}} |\hat{\mathbf{z}}_\mathbf{j}^\mathbf{in}| + \sum_{\mathbf{k=1}}^{\mathbf{r}} |\hat{\mathbf{z}}_\mathbf{k}^\mathbf{pr}|$ and $\alpha = \max(\alpha_h)$ with $\boldsymbol{\alpha}_\mathbf{h} = \frac{\mathbf{z_h^{out}}}{\hat{\mathbf{z}}_\mathbf{h}^\mathbf{out}}$ then $\widehat{\mathbf{Z}}^* = \boldsymbol{\alpha}\widehat{\mathbf{Z}}$ provide $MAE(\widehat{\mathbf{Z}}^*) = MAE(\widehat{\mathbf{Z}})$.

Proof:

According to (9)
$\sum_i^n |e_i| - \sum_i^n |e^*_i| = (1-\alpha)\{\{\sum_{h=1}^p |\hat{z}_h^{out}| + \sum_{l=1}^s |\hat{z}_l^{cr}|\} - \{\sum_{j=1}^q |\hat{z}_j^{in}| + \sum_{k=1}^r |\hat{z}_k^{pr}|\}\}$
so $\sum_i^n |e_i| - \sum_i^n |e^*_i| = 0$, or $MAE(\widehat{Z}) = \sum_i^n |e_i| - \sum_i^n |e^*_i| = MAE(\widehat{Z}^*)$ ▓

According to Consequence 2, giving scale factor can be repeated, until we have $\sum_{h=1}^p |\hat{z}_h^{out}| + \sum_{l=1}^s |\hat{z}_l^{cr}| \le \sum_{j=1}^q |\hat{z}_j^{in}| + \sum_{k=1}^r |\hat{z}_k^{pr}|$.

If a model is included in the compressed category, it is necessary to stretch to get better accuracy, which minimize error.

Definition 5

Given Z the result of the transformation of the realization of a time series, $\widehat{\mathbf{Z}}$ is the prediction model with outer $\mathbf{z_h^{out}}$ and $\hat{\mathbf{z}}_\mathbf{h}^\mathbf{out}$, inner $\mathbf{z_j^{in}}$ and $\hat{\mathbf{z}}_\mathbf{j}^\mathbf{in}$, precise $\mathbf{z_k^{pr}}$ and $\hat{\mathbf{z}}_\mathbf{k}^\mathbf{pr}$ as well as crossover $\mathbf{z_l^{cr}}$ and $\hat{\mathbf{z}}_\mathbf{l}^\mathbf{cr}$,

A model $\widehat{\mathbf{Z}}$ is said compressed if $\sum_{\mathbf{j=1}}^{\mathbf{q}} |\hat{\mathbf{z}}_\mathbf{j}^\mathbf{in}| > \sum_{\mathbf{h=1}}^{\mathbf{p}} |\hat{\mathbf{z}}_\mathbf{h}^\mathbf{out}| + \sum_{\mathbf{l=1}}^{\mathbf{s}} |\hat{\mathbf{z}}_\mathbf{l}^\mathbf{cr}| + \sum_{\mathbf{k=1}}^{\mathbf{r}} |\hat{\mathbf{z}}_\mathbf{k}^\mathbf{pr}|$

Stretching will result in increasing the absolute price number of errors in the points *outer* $\hat{z}_h^{out}$, *crossover* $\hat{z}_l^{cr}$ and *precise* $\hat{z}_k^{pr}$ which are beyond the center line scale, but decrease the absolute price number of errors in the points *inner* $\hat{z}_j^{in}$. Thus, a model stretching can be done to minimize the error if the absolute price number of errors in the points *inner* $\hat{z}_j^{in}$, More than the absolute price number of the error on all other points.

The idea of strecthing the model is to give a scale factor to squeeze point $\hat{z}_j^{in}$ and point z_j^{in} which has the shortest distance, this is conducted to ensure that the error of the entire point outer $\hat{z}_j^{in}$ will shrink. Selecting a point $\hat{z}_j^{in}$ and point z_j^{in} which has the closest distance, then choose $\frac{z_j^{in}}{\hat{z}_j^{in}}$ the smallest, because the points $\hat{z}_j^{in}$ always closer than z_j^{in} towards the center of the scale.

Theorem 3

Given Z the result of the transformation of the realization of a time series, $\widehat{\boldsymbol{Z}}$ is the prediction model with outer $\boldsymbol{z_h^{out}}$ and $\hat{\boldsymbol{z}}_{\boldsymbol{h}}^{\boldsymbol{out}}$, inner $\boldsymbol{z_j^{in}}$ and $\hat{\boldsymbol{z}}_{\boldsymbol{j}}^{\boldsymbol{in}}$, precise $\boldsymbol{z_k^{pr}}$ and $\hat{\boldsymbol{z}}_{\boldsymbol{k}}^{\boldsymbol{pr}}$ as well as crossover $\boldsymbol{z_l^{cr}}$ and $\hat{\boldsymbol{z}}_{\boldsymbol{l}}^{\boldsymbol{cr}}$,

If $\sum_{j=1}^q |\hat{z}_j^{in}| > \sum_{h=1}^p |\hat{z}_h^{out}| + \sum_{l=1}^s |\hat{z}_l^{cr}| + \sum_{k=1}^r |\hat{z}_k^{pr}|$ *and* $\beta = \min(\beta_j)$ *With* $\beta = \frac{z_j^{in}}{\hat{z}_j^{in}}$

then $\widehat{Z}^* = \beta\widehat{Z}$ *provide* $MAE(\widehat{Z}^*) < MAE(\widehat{Z})$.

Proof:

Made $\beta_j = \frac{z_j^{in}}{\hat{z}_j^{in}}$. Because for every *j* applicable $|\hat{z}_j^{in}| < |z_j^{in}|$ then

$\beta_j > 1$ (10)

Taken $\beta = \min(\beta_j : j = 1, 2, \dots p)$, then for each $j = 1, 2, \dots q$ applicable

$|\beta\hat{z}_j^{in}| \le |\beta_j\hat{z}_j^{in}| = |z_j^{in}|$ (11)

Given
$\hat{z}_h^{out*} = \beta\hat{z}_h^{out}$, $e_h^{out} = \hat{z}_h^{out} - z_h^{out}$, $e_h^{out*} = \hat{z}_h^{out*} - z_h^{out}$
$\hat{z}_j^{in*} = \beta\,\hat{z}_j^{in}$, $e_j^{in} = \hat{z}_j^{in} - z_j^{in}$, $e_j^{in*} = \hat{z}_j^{in*} - z_j^{in}$,
$\hat{z}_k^{pr*} = \beta\,\hat{z}_k^{pr}$, $e_k^{pr} = \hat{z}_k^{pr} - z_k^{pr}$, $e_k^{pr*} = \hat{z}_k^{pr*} - z_k^{pr}$,
$\hat{z}_l^{cr*} = \beta\,\hat{z}_l^{cr}$, $e_l^{cr} = \hat{z}_l^{cr} - z_l^{cr}$, $e_l^{cr*} = \hat{z}_l^{cr*} - z_l^{cr}$,

then

$\sum_{h=1}^p |e_h^{out*}| = \sum_{h=1}^p |\hat{z}_h^{out*} - z_h^{out}| = \sum_{h=1}^p |\beta\hat{z}_h^{out} - z_h^{out}|$ (12)
$\sum_{j=1}^q |e_j^{in*}| = \sum_{j=1}^q |\hat{z}_j^{in*} - z_j^{in}| = \sum_{j=1}^q |\beta\hat{z}_j^{in} - z_j^{in}|$ (13)
$\sum_{k=1}^r |e_k^{pr*}| = \sum_{k=1}^r |\hat{z}_k^{pr} - z_k^{pr}| = \sum_{k=1}^r |\beta\hat{z}_k^{pr} - z_k^{pr}|$ (14)
$\sum_{l=1}^s |e_l^{cr*}| = \sum_{l=1}^s |\hat{z}_l^{cr*} - z_l^{cr}| = \sum_{\mathbf{l=1}}^{\mathbf{s}} |\boldsymbol{\beta}\hat{\boldsymbol{z}}_{\boldsymbol{l}}^{\boldsymbol{cr}} - \boldsymbol{z}_{\boldsymbol{l}}^{\boldsymbol{cr}}|$ (15)

Given $\sum_{i=1}^n |e_i^*|$ with
$\sum_{i=1}^n |e_i^*| = \sum_{h=1}^p |e_h^{out*}| + \sum_{j=1}^q |e_j^{in*}| + \sum_{k=1}^r |e_k^{r*}| + \sum_l^s |e_l^{cr*}|$

Because (12), (13), (14) and (15) then

$\sum_{i=1}^{n}|e_i^*|=\sum_{h=1}^{p}\left|\beta\hat{z}_h^{out}-z_h^{out}\right|+\sum_{j=1}^{q}\left|\beta\hat{z}_j^{in}-z_j^{in}\right|+\sum_{k=1}^{r}\left|\beta\hat{z}_k^{pr}-z_k^{pr}\right|+\sum_{l=1}^{s}\left|\boldsymbol{\beta\hat{z}_l^{cr}-z_l^{cr}}\right|$

Because (10), (11), Definition 2, dan Definition **3**, then

$\sum_{i=1}^{n}|e_i^*|=\sum_h^p\left(\left|\beta\hat{z}_h^{out}\right|-\left|z_h^{out}\right|\right)+\sum_j^q\left(\left|z_j^{in}\right|-\left|\beta\hat{z}_j^{in}\right|\right)+\sum_{k=1}^{r}\left(\left|\beta\hat{z}_k^{pr}\right|-\left|z_k^{pr}\right|\right)+\sum_{l=1}^{s}\left(\left|\beta\hat{z}_l^{cr}\right|+\left|z_l^{cr}\right|\right)$

$=\left(\sum_h^p\left|\beta\hat{z}_h^{out}\right|-\sum_h^p\left|z_h^{out}\right|\right)+\left(\sum_j^q\left|z_j^{in}\right|-\sum_j^q\left|\beta\hat{z}_j^{in}\right|\right)+\left(\sum_{k=1}^{r}\left|\beta\hat{z}_k^{pr}\right|-\sum_{k=1}^{r}\left|z_k^{pr}\right|\right)+\left(\sum_{l=1}^{s}\left|\beta\hat{z}_l^{cr}\right|+\sum_{l=1}^{s}\left|z_l^{cr}\right|\right)$ (16)

Given $\sum_{i=1}^{n}|e_i|$ and $e_h^{out}=\hat{y}_h^{out}-y_h^{out}$, $e_j^{in}=\hat{y}_j^{in}-y_j^{in}$, $e_k^{pr}=\hat{y}_k^{pr}-y_k^{pr}$, $e_l^{cr}=\hat{y}_l^{cr}-y_l^{cr}$. According to Definition 2, and Definition 3, then e_i can be partitioned into *outer, inner, precise* and *crosover*, so

$\sum_{i=1}^{n}|e_i|=\sum_{h=1}^{p}\left|e_h^{out}\right|+\sum_{j=1}^{q}\left|e_j^{in}\right|+\sum_{k=1}^{r}\left|e_k^{pr}\right|+\sum_l^s\left|e_l^{cr}\right|$

$=\sum_{h=1}^{p}\left|\hat{z}_h^{out}-z_h^{out}\right|+\sum_{j=1}^{q}\left|\hat{z}_j^{in}-z_j^{in}\right|+\sum_{k=1}^{r}\left|\hat{z}_k^{pr}-z_k^{pr}\right|+\sum_{l=1}^{s}\left|\boldsymbol{\hat{z}_l^{cr}-z_l^{cr}}\right|$

Due to the properties on Definition 2, dan Definition 3, then

$\sum_{i=1}^{n}|e_i|=\sum_h^p\left(\left|\hat{z}_h^{out}\right|-\left|z_h^{out}\right|\right)+\sum_j^q\left(\left|z_j^{in}\right|-\left|\hat{z}_j^{in}\right|\right)+\sum_{k=1}^{r}\left(\left|\hat{z}_k^{pr}\right|-\left|z_k^{pr}\right|\right)+\sum_{l=1}^{s}\left(\left|\hat{z}_l^{cr}\right|+\left|z_l^{cr}\right|\right)$

$=\left(\sum_h^p\left|\hat{z}_h^{out}\right|-\sum_h^p\left|z_h^{out}\right|\right)+\left(\sum_j^q\left|z_j^{in}\right|-\sum_j^q\left|\hat{z}_j^{in}\right|\right)+\left(\sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|-\sum_{k=1}^{r}\left|z_k^{pr}\right|\right)+\left(\sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right|+\sum_{l=1}^{s}\left|z_l^{cr}\right|\right)$ (17)

Given $\sum_i^n|e_i|-\sum_i^n|e_i^*|$ According to (16) and (17) then

$\sum_i^n|e_i|-\sum_i^n|e_i^*|=\left(\sum_h^p\left|\hat{z}_h^{out}\right|-\sum_h^p\left|z_h^{out}\right|\right)+\left(\sum_j^q\left|z_j^{in}\right|-\sum_j^q\left|\hat{z}_j^{in}\right|\right)+\left(\sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|-\sum_{k=1}^{r}\left|z_k^{pr}\right|\right)+\left(\sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right|+\sum_{l=1}^{s}\left|z_l^{cr}\right|\right)-\left(\sum_h^p\left|\beta\hat{z}_h^{out}\right|-\sum_h^p\left|z_h^{out}\right|\right)-\left(\sum_j^q\left|z_j^{in}\right|-\sum_j^q\left|\beta\hat{z}_j^{in}\right|\right)-\left(\sum_{k=1}^{r}\left|\beta\hat{z}_k^{pr}\right|-\sum_{k=1}^{r}\left|z_k^{pr}\right|\right)-\left(\sum_{l=1}^{s}\left|\beta\hat{z}_l^{cr}\right|+\sum_{l=1}^{s}\left|z_l^{cr}\right|\right)$

According to the commutative properties, then

$\sum_i^n|e_i|-\sum_i^n|e_i^*|=\left(\sum_h^p\left|\hat{z}_h^{out}\right|-\sum_h^p\left|z_h^{out}\right|\right)-\left(\sum_h^p\left|\beta\hat{z}_h^{out}\right|-\sum_h^p\left|z_h^{out}\right|\right)+\left(\sum_j^q\left|z_j^{in}\right|-\sum_j^q\left|\hat{z}_j^{in}\right|\right)-\left(\sum_j^q\left|z_j^{in}\right|-\sum_j^q\left|\beta\hat{z}_j^{in}\right|\right)+\left(\sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|-\sum_{k=1}^{r}\left|z_k^{pr}\right|\right)-\left(\sum_{k=1}^{r}\left|\beta\hat{z}_k^{pr}\right|-\sum_{k=1}^{r}\left|z_k^{pr}\right|\right)+\left(\sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right|+\sum_{l=1}^{s}\left|z_l^{cr}\right|\right)-\left(\sum_{l=1}^{s}\left|\beta\hat{z}_l^{cr}\right|+\sum_{l=1}^{s}\left|z_l^{cr}\right|\right)$

$=\left(\sum_{h=1}^{p}\left|\hat{z}_h^{out}\right|-\sum_{h=1}^{p}\left|\beta\hat{z}_h^{out}\right|\right)-\left(\sum_{j=1}^{q}\left|\hat{z}_j^{in}\right|-\sum_{j=1}^{q}\left|\beta\hat{z}_j^{in}\right|\right)+\left(\sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|-\sum_{k=1}^{r}\left|\beta\hat{z}_k^{pr}\right|\right)+\left(\sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right|-\sum_{l=1}^{s}\left|\beta\hat{z}_l^{cr}\right|\right)$

According to the distributive and associative properties, then

$\sum_i^n|e_i|-\sum_i^n|e_i^*|=\sum_{h=1}^{p}\left(\left|\hat{z}_h^{out}\right|-\left|\beta\hat{z}_h^{out}\right|\right)-\sum_{j=1}^{q}\left(\left|\hat{z}_j^{in}\right|-\left|\beta\hat{z}_j^{in}\right|\right)+\sum_{k=1}^{r}\left(\left|\hat{z}_k^{pr}\right|-\left|\beta\hat{z}_k^{pr}\right|\right)+\sum_{l=1}^{s}\left(\left|\hat{z}^{cr}{}_l\right|-\left|\beta\hat{z}_l^{cr}\right|\right)$

$=\sum_{h=1}^{p}(1-\beta)\left|\hat{z}_h^{out}\right|-\sum_{j=1}^{q}(1-\beta)\left|\hat{z}_j^{in}\right|+\sum_{k=1}^{r}(1-\beta)\left|\hat{z}_k^{pr}\right|+\sum_{l=1}^{s}(1-\beta)\left|\hat{z}_l^{cr}\right|$

$=(1-\beta)\left\{\sum_{h=1}^{p}\left|\hat{z}_h^{out}\right|-\sum_{j=1}^{q}\left|\hat{z}_j^{in}\right|+\sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|+\sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right|\right\}$

$=(1-\beta)\left\{\left\{\sum_{h=1}^{p}\left|\hat{z}_h^{out}\right|+\sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right|+\sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|\right\}-\left\{\sum_{j=1}^{q}\left|\hat{z}_j^{in}\right|\right\}\right\}$ (18)

Because, the terms on Theorem 3 is

$\left\{\sum_{h=1}^{p}\left|\hat{z}_h^{out}\right|+\sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right|+\sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|\right\}<\sum_{j=1}^{q}\left|\hat{z}_j^{in}\right|$

and $\beta > 1$, then $\sum_i^n|e_i|-\sum_i^n|e_i^*| > 0$, so

$MAE(\hat{Z}^*)=\frac{1}{n}\sum_i^n|e_i^*|<\frac{1}{n}\sum_i^n|e_i|=MAE(\hat{Z})$. ∎

Assurance that the resulting MAE is the smallest, is not given by Theorem 3. Whereby, if the giving scale factor is already done, needs to be double check on the condition of the theorem. If the condition is met, the giving scale factor can be done again.

Consequence 4

Given Z the result of the transformation of the realization of a time series, $\hat{Z}$ is the prediction model with outer z_h^{out} and $\hat{z}_h^{out}$, inner z_j^{in} and $\hat{z}_j^{in}$, precise z_k^{pr} and $\hat{z}_k^{pr}$ as well as crossover z_l^{cr} and $\hat{z}_l^{cr}$,

If $\sum_{j=1}^{q}\left|\hat{z}_j^{in}\right|=\sum_{h=1}^{p}\left|\hat{z}_h^{out}\right|+\sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right|+\sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|$ and $\beta = min(\beta_j)$ with $\beta=\frac{z_j^{in}}{\hat{z}_j^{in}}$

then $\hat{Z}^*=\beta\hat{Z}$ provide $MAE(\hat{Z}^*)=MAE(\hat{Z})$.

Proof:

The terms on Consequence 4 is $\sum_{j=1}^{q}\left|\hat{z}_j^{in}\right|=\sum_{h=1}^{p}\left|\hat{z}_h^{out}\right|+\sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right|+\sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|$, and according to (18)

$\sum_i^n|e_i|-\sum_i^n|e_i^*|=(1-\beta)\left\{\left\{\sum_{h=1}^{p}\left|\hat{z}_h^{out}\right|+\sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right|+\sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|\right\}-\left\{\sum_{j=1}^{q}\left|\hat{z}_j^{in}\right|\right\}\right\}$

then $\sum_i^n|e_i|-\sum_i^n|e_i^*|=0$, so

$MAE(\hat{Z})=\sum_i^n|e_i|=\sum_i^n|e^*{}_i|=MAE(\hat{Z}^*)$ ∎

Based on Consequence 4, then if the process of scaling factor is still producing $\sum_{j=1}^{q}\left|\hat{z}_j^{in}\right|>\sum_{h=1}^{p}\left|\hat{z}_h^{out}\right|+\sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right|+\sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|$ then the process of giving scale factor performed again till obtain $\sum_{j=1}^{q}\left|\hat{z}_j^{in}\right|\leq\sum_{h=1}^{p}\left|\hat{z}_h^{out}\right|+\sum_{l=1}^{s}\left|\hat{z}_l^{cr}\right|+\sum_{k=1}^{r}\left|\hat{z}_k^{pr}\right|$.

3. Conclusion

A fuzzy model can be relatively stretched or compressed to the data. If a model has the stretches, the MAE can be reduced by compressing the model. Likewise, if a model is more compressible than the data that is available, then the model can be stretched to minimize MAE

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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