ABSTRACT

This study aims to explore and describe the understanding of prospective mathematical language in mathe-matics teachers in learning. A qualitative approach with a descriptive type is used for this purpose. Three Mathematics Teacher Professional Education (TPEP) students at the University of Muhammadiyah Malang who are currently pursuing Field Experience Programs (FEP) in high schools are used as research subjects. Subjects were selected based on the Achievement Index (IP) obtained namely, high, medium, and low. Data obtained through the study of the Learning Implementation Plan (LIP) and Student Worksheet (SW) documents, observations during the learning process, and interviews. The data analysis technique is done by data reduction, data presentation, and conclusion drawing. The results showed that the understanding of the mathematical language of subjects with high and moderate IPs was classified as very good while the understanding of the mathematical language of subjects with low IP was classified as good.

INTRODUCTION

The term language is defined as words in which the pronunciation and methods used are understood by a community [1]. Mathematics has its own language [2]. The language of mathematics is the system used by mathematicians to communicate mathematical ideas among themselves. This language contains natural language with the use of technical terms and grammatical conventions that are specific to mathematical discourse, coupled with symbolic notations that are very specific to mathematical formulas [3]. Specifically, concerning mathematical language, the ability to use words, explain, justify, and communicate mathematically is important for the overall development of mathematical abilities [1, 4].

Language in learning mathematics is understood as a source for mathematical thinking in mathematical activities with other people as well as activities to interpret mathematics in other situations [5]. The main function of language in teaching mathematics is to enable teachers and students to communicate mathematical knowledge appropriately [6].

Mathematical languages consist of natural languages using technical terms and grammatical conventions that are specific to mathematical discourse, plus symbolic notations that are very specific to mathematical formulas. The mathematical language syntax includes a list of symbols, configuration of rules for constructing language patterns, axioms, deductive systems, and theorems. Elements in mathematical language include symbols, concepts, definitions, and theorems [3]. Mathematical terms and symbols are clearly defined. Each statement in mathematical language has only one meaning or is not ambiguous. Each mathematical pattern has one structure that is determined by operational rules [3].

Many advanced mathematical sentences have complicated structures that are easy to understand if one knows some basic mathematical terms [7]. Good mathematical language skills require a strong base of vocabulary knowledge; flexibility; fluency and ability to understand numbers, symbols, and sentences [1]. Fact, students have difficulty in understanding these elements including using definitions of mathematical terms and related concepts [6], formulating mathematical models, transforming problems into mathematical equations [8]. Students have difficulty in determining the operations that must be carried out to find solutions because they are unable to make appropriate mathematical models [9].

Factors that cause students difficulty understanding the language of mathematics are complex forms of mathematics and students feel less familiar with the words learned [8]. Mathematical language learning activities will be difficult because of the abstractness of the objects discussed and their consequences affect the difficulty of expressing the things discussed [6]. Thus the teacher first needs to have good mathematical language skills. This is because teacher knowledge will have an impact on student achievement [10]. Teachers who have good mathematical language skills will be able to better teach students how to use sentences in mathematics, symbols, and multi-representations [5].
Mathematics Language Understanding Of Teachers’ Candidate In Mathematics Learning

Teachers have an important role in learning because teacher performance will affect student achievement and student involvement in the learning process [11]. In realizing the goals of mathematics learning, teachers need to use structured languages, namely technical vocabulary and symbolism that can be understood by students [6]. This is because mathematics is fundamentally procedural that focuses on techniques that involve numbers, symbols, and equations. Patterns and relationships between concepts consisting of symbols and equations are real problems that must be understood in depth. Therefore, mathematical logic and grammar are very important to understand [12].

Teachers still have difficulty in explaining mathematical concepts in the form of mathematical language [13]. Even though the teacher claimed to have mastered the concept but was confused to explain it. Teachers also have difficulties in expressing mathematical problems in mathematical symbols or sentences [14]. This is a problem in learning mathematics because teachers need to be able to give mathematical meaning to students through the ability to read, write, and discuss mathematical concepts [5]. The challenge for mathematics teachers is not only the complexity of the mathematical concepts taught and the various prerequisite concepts needed but also the sophistication of the semiotic system that exists in mathematics [15]. Therefore, the preparation of competencies for prospective teachers or teachers is very important.

The teacher professional education program (TPEP) is an effort by the Indonesian government to prepare professional teachers to realize the goals of national education. TPEP is an educational program held to prepare graduates of Education and Undergraduate/Non-Education IV who have the talent and interest to become teachers to fully master teachers’ competencies under national education standards so that they can obtain a certificate of professional educators [16] with a period of education for one semester or two semesters. TPEP Mathematics students are prospective mathematics teachers and will become mathematics teachers after they are declared graduated and obtain educator certificates. The intended teacher competence consists of personality, social, professional, and pedagogical competencies [17]. The development of prospective mathematics teachers using appropriate mathematical language is an important aspect of teacher education [18, 19].

Professional and pedagogical competencies related to the mastery of the content of the material and how to teach them are two very important competencies for prospective teachers [20]. Prospective teachers must have the ability to understand the connection of one notation to another or from one procedure to another [13] which is one of the professional competencies besides using clear language to express the reasons behind mathematical procedures [13]. Through mathematical vocabulary and definitions, students explore the concepts involved and learn to mean [1]. Mathematical language teaching to students helps teachers identify more clearly what is the source of difficulty and helps them understand how to make the mathematical language more meaningful to students [21].

Mathematical language consists of semiotic and grammatical/syntactic aspects [22]. Semiotics is the study of signs, which can be in the form of words, images, sounds, gestures, and objects [23]. Syntax studies the relationship between one word and another, or other elements as a whole. The syntactic unit consists of words, phrases, clauses, sentences, and discourse [24]. Semiotic aspects include mathematical symbol notation and graphics or visual displays. Whereas the grammatical effect consists of technical vocabulary and implicit logical relationships [22]. The syntactic unit observed in this study focuses on vocabulary and sentences used.

Symbols have characteristics that can be viewed from aspects of materiality, syntax, and meaning. Materials symbols can be in the form of Latin letters, operators, and physical attributes. Besides, the symbol syntax states the position and conventions associated with it. For example, the “=” sign is used to express the same value on both sides. In meaning, symbols are placed in the domain that states the context of a problem [25]. The meaning of mathematical symbols is also expressed by [26] which states that mathematical symbols consist of three categories, namely letters, figures, and other templates that combine letters and other figures in a two-dimensional form. Symbols in the form of letters such as \( a, A, \alpha, \beta, \pi \). The symbols in the form of figures such as \( +, %, \sqrt{}, f, = \). Symbols in the form of combined templates such as \( f(x, y) \) and \( dy \) [27].

Graphs are an example of visual appearance in mathematics. Several other visual displays that can be used in understanding mathematical concepts. In mathematics, the choice of visual appearance is related to what is to be expressed [28]. One example of a visual display such as images of wake-up flat and wake up space.

A word is a small unit of a sentence [29]. Sentences can be interpreted in three understandings, namely sentences as expressions with specific structures or forms, sentences as expressions with specific contexts or meanings, and sentences as specific uses [30]. Sentences in mathematics can be definitions, theorems, or other statements containing elements of logic. Logic sentences can be in the form of implications, bi-implications, negations, or sentences of office. Mathematical logic sentences must be proven true [31]. Thus, a sentence has an implicit logical relationship if the sentence is following the rules of logic.

Some of the previous studies relating to the language of mathematics are [32] which states that mathematics becomes difficult to understand because it contains terms and symbols that are difficult to understand. Therefore, teaching about the language of mathematics will increase the effectiveness of learning
mathematics. The same thing was also expressed [1] that learning of terms in mathematics became something very important. In learning mathematics, students have difficulty in translating terms in mathematics [9] as well as making mathematical models of the given problems [8]. The mathematical language skills of prospective mathematics teachers (mathematics education students) [33].

Because naturally mathematical content develops from less complex skills to very complex ones and the importance of the teacher's role in helping students use mathematical language effectively, mastering the mathematical language and the ability to teach it to prospective teachers is very important. Therefore, this study aims to explore the understanding of mathematical language by prospective mathematics teachers who are undergoing professional education teachers. The problems examined in this study are: how is the understanding of the mathematical language of prospective mathematics teachers who are attending professional education teachers?

THE RESEARCH METHODS

DESIGN

A qualitative approach with an explorative descriptive type was used in this study, because the aim of this study is to explore information about understanding mathematical language of prospective mathematics teachers who were obtained in depth and describe a situation as it is, without giving any action [34, 35].

SUBJECTS

The subjects of this study were three prospective mathematics teachers who were attending professional education teachers at the University of Muhammadiyah Malang and taking the Field Experience Program (FEP) at partner schools, namely public high schools 8 and 9 in Malang. Subjects are undergraduate mathematics education graduates selected based on the Achievement Index (IP) obtained. Subjects with high IP (S1), medium IP (S2), and with low IP (S3).

DATA COLLECTION

The research data in the form of understanding of the mathematical language were obtained through a review of the Learning Implementation Plan (LIP) and Student Worksheet (SW) documents made by prospective teachers, observations during the learning process in class, and interviews. Data collection was carried out on each subject for three meetings, each meeting using 2 x 40 minutes. Method triangulation and time triangulation are used to obtain valid data [35, 36].

DATA ANALYSIS

Data analysis techniques in this study use an interactive model [34, 35] namely: 1) data reduction includes the activities of selecting, simplifying and transforming rough data from document review, interviews, and observations conducted during learning activities; 2) data presentation, namely compiling information in the form of narrative text and calculation results tables that explain the understanding of mathematical language and the pedagogical abilities of teacher candidates used in analyzing data; 3) drawing conclusions namely verification of data obtained during the research process. Mathematical language understanding of prospective mathematics teachers is classified by the categories, very good if 75% ≤ U ≤ 100%, good if 50% ≤ U < 75%, not good if 25% ≤ U < 50%, and not very good if 0% ≤ U < 25%, where U is Understanding Mathematical Languages [37].

RESULTS

Understanding the mathematics language of prospective teachers in terms of the use of semiotic lists and also grammatical patterns are presented in general and deepened discussed in each subject. Percentage of understanding of the mathematics language of prospective teachers is obtained through the results of the document review, namely the LIP and SW used at three meetings, the results of observations in class during the learning activities taking place, and interviews. The results of the document review regarding the overall understanding of the mathematics language of prospective teachers are presented in Table 1.

| Table 1. Understanding Mathematics Language of Teachers Prospective |
|-------------------------|-----------------|-----------------|-----------------|-----------------|
|                         | LIP Meeting 1   | LIP Meeting 2   | LIP Meeting 3   | Av. Categ.      |
| Subjects               | Right | Right | Right | Right | Right | Right | Right | Right | Right |
| S1                     | 74% | 5%, 77% | 22%, 5% | 73% | 5%, 26% | 75% | Very good |
| S2                     | 77%, 53% | 22%, 5% | 75% | 26%, 5% | 76%, 10% | 76% | Very good |
| S3                     | 60%, 19% | 26%, 30% | 39%, 74% | 57%, 52% | 42%, 48% | 59%, 26% | Good |

UNDERSTANDING MATHEMATICS LANGUAGE S1

Understanding S1 mathematical language is classified as very good. This is seen in 75% of the use of appropriate mathematical language. Some mathematical concepts and ideas are also well understood. One example is the following interview excerpt.

Researcher : What do you think the symbols of mathematics mean?
S1 : Symbols in mathematics are an agreement in representing a mathematical definition or formula.

Researcher : What about graphs? How do you view the graph?
S1 : The graph is to presents data. On the graph, there are 2 axes, the X-axis and the Y-axis. We have to know what X-axis represents what data and Y-axis represent what data.
S1 understands symbols as a sign which is an agreement in representing a mathematical idea. Also, S1 understands graphs as displays used to present data. Sometimes S1 has difficulty in explaining the definition to students. This is because explaining new vocabulary to students must use simple and clear sentence structures so that students can understand it well. Some mathematical languages can be used well by S1 but there are also some mistakes. The proper use of mathematical language, for example, is described in Table 2.

### Table 2. Accuracy of S1 in Using Mathematics Language

<table>
<thead>
<tr>
<th>No.</th>
<th>Appropriate semiotic/grammatical pattern</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>If we add up the difference in data with the average, we get $\sum (x_i - \bar{x}) = 0$</td>
<td>It contains semiotic elements (symbols) and grammatical patterns (use of technical vocabulary). Representation of words into mathematical sentence form. The notation and symbols used are exact. The vocabulary used is precise and logical.</td>
</tr>
<tr>
<td>2.</td>
<td>Projection $DE$ in the field $BDHF$ is $DE'$</td>
<td>It contains semiotic elements in the form of visual appearance (pictures). The visual display in the form of a figure of space and its projections are presented appropriately. The images presented represent the conditions given.</td>
</tr>
</tbody>
</table>

From the semiotic aspect, S1 has used several symbols appropriately such as the sigma symbol ($\sum$) and also the average ($\bar{x}$). Besides, from the aspect of grammatical patterns the vocabulary used is also appropriate and has a logical relationship. Visual displays such as histograms and projected images of space constructions are also well presented and represent the conditions given. Besides, there are some errors in the understanding of the S1 mathematics language as presented in Table 3.

### Table 3. Errors of S1 in the Use of Mathematics Language

<table>
<thead>
<tr>
<th>No.</th>
<th>Semiotic/grammatical patterns that are not quite right</th>
<th>Correction</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>The use of colons that are not in line with the math sentence (...)</td>
<td>Use a triangle which is in line with the math sentence (...)</td>
<td>Mistaken use of symbols</td>
</tr>
<tr>
<td>2.</td>
<td>$f_i = \text{frequency}$ $f_i = \text{frequency of the i-th data}$</td>
<td>Information on notation is incomplete</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>the point of the i-th class is $x_i$, the midpoint of the i-th class is $x_i$</td>
<td>Information on notation is incomplete</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>The line segment AB denoted by $AB$ $AB$</td>
<td>Mistaken use of symbols</td>
<td></td>
</tr>
</tbody>
</table>

Table 3 shows that there are some errors in the use of mathematics language by S1. The mistake in terms of semiotics is the mistake of using symbols. Besides, there are also errors in terms of grammatical patterns, namely the vocabulary used to explain the meaning of symbols is not right.

### UNDERSTANDING MATHEMATICS LANGUAGE S2

Understanding the S2 mathematics language is classified as very good. S2 gained the largest percentage compared to other subjects, reaching 76.19%. Some mathematical concepts and ideas are also well understood. One example is the following interview excerpt:

*Researcher*: What do you think the mathematical symbols mean?

*S2*: Mathematical symbols are things that express meaning about mathematical ideas.

*Researcher*: What is the meaning of the graph?

*S2*: Graphs are used to present data.

The mathematical symbol according to S2 is something that expresses the meaning of mathematical ideas. Understanding the S2 language is better than other subjects. Besides, S2 also admitted that he had no difficulty in explaining a mathematical concept to students. Some mathematical languages can be used well by S2, but there are also some mistakes. The proper use of mathematical language, for example, is described in Table 4.

### Table 4. Accuracy of S2 in Using Mathematics Language

<table>
<thead>
<tr>
<th>No.</th>
<th>Semiotic/grammatical patterns that are appropriate</th>
<th>Description</th>
</tr>
</thead>
</table>

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Table 5 shows that there were some errors in the use of mathematics by S2. These errors include the use of inconsistent notations and the use of symbols. Besides, there is also a mistake in the presentation of the graph because the distance between the number lines on the coordinate axis is not consistent (the scale is inconsistent).

### 1.1 Understanding Mathematics language S3

The mathematical language understanding of S3 is quite good with the acquisition of a percentage of 59.26%. Not much different from the views of other subjects, S3 states that mathematical symbols are meanings/signs that state something that must be interpreted or done. This can be seen in the following interview excerpt.

**Researcher:** What do you think is the meaning of mathematical symbols?

**S3:** I think the mathematical symbol is a sign of an idea or thing that has to be done. For example, the addition sign (+) means we have to do the addition operation of two or more numbers.

**Researcher:** How about the graph?

**S3:** The graph to state the relationship between data

S3 sometimes experience confusion in using the right mathematical language and following the concept. For example, when teaching integers sometimes S3 uses the command word "forward/backward" and sometimes uses the word "right/left". This can lead to a misunderstanding of the basic concepts of integers. S3 is also often mistaken in distinguishing "plus" or "minus" sign operations and "positive" or "negative" numbers. This indicates that S3 has not differentiated the basic concepts of number values and number operations. This is important to note because number values and number operations have different meanings.

Some mathematical languages can be used well by S2 but there are also some mistakes. The proper use of mathematical language, for example, is described in Table 6.

### Table 5. MISTAKES OF S2 IN THE USE OF MATHEMATICS LANGUAGE

<table>
<thead>
<tr>
<th>No.</th>
<th>Semiotic/grammatical patterns that are not quite right</th>
<th>Correction</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>The frequency the class uses the notation ( f_i ) sometimes uses ( f )</td>
<td>Use the notation ( f_i ) to express the class frequency</td>
<td>Use of inconsistent notations</td>
</tr>
<tr>
<td>2.</td>
<td>( \cdots + \cdots + \cdots + \cdots + \cdots + \cdots + \cdots ) ( = 7,5 )</td>
<td>Use point four (( \cdots )) which states fill in the points with the right answer</td>
<td>Mistaken use of symbols</td>
</tr>
</tbody>
</table>

### Table 6. ACCURACY OF S3 IN USING MATHEMATICS LANGUAGE

<table>
<thead>
<tr>
<th>No.</th>
<th>Semiotic/grammatical patterns that are appropriate</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>The form ((x-a)(x-b)) is other writing from ( x^2 - (a+b)x + ab ) which means the form is a tribe of many degrees 2.</td>
<td>The vocabulary used is precise and has an implicit logical relationship to explain the degree of a multi-syllable</td>
</tr>
<tr>
<td>2.</td>
<td>Two is not a factor of ( f(x) = 2x^3 - x^2 - 7x + 6 ), because after all the possible ( h ) values are substituted, no resulting ( h ) values</td>
<td>The vocabulary used is precise and has an implicit logical relationship to explain the concept of theorem factor.</td>
</tr>
</tbody>
</table>
Some of the examples above state the accuracy of the use of mathematical language by S3 in terms of both semiotic and grammatical patterns. From the semiotic aspect, S2 explains the concept of many tribal degrees and factor theorems with languages that have a logical relationship. From the aspect of grammatical patterns, S2 also presents the horner chart appropriately. In addition, there are also some mistakes in understanding the S2 mathematics language as presented in Table 7.

Table 7. Mistakes of S3 in the Use of Mathematics Language

<table>
<thead>
<tr>
<th>No.</th>
<th>Inappropriate words/phrases/sentences</th>
<th>Correct words/phrases/sentences</th>
<th>Explanations</th>
</tr>
</thead>
</table>
| 1.  | $x = 1$  
$\rightarrow (1)^3$  
$- 11(1)^2$  
$+ 30(1) - 8$  
$= 12$  

$\text{if } x = 1 \text{ then}$  
$f(x) = (1)^3$  
$- 11(1)^2$  
$+ 30(1)$  
$- 8$  
$= 12$  |
| 2.  | $(z^4 + 5z^3) \div (z + 3)$ produce $z^3$ as the initial term of the quotient  

$\text{The use of vocabulary that can cause confusion in understanding concepts}$  |
| 3.  | Polynomial division $h(y)$ by $(px + q)$  

$\text{polynomial division}$  
$h(y) \div (px + q)$  

$\text{Writing error}$  |
| 4.  | Sometimes teachers use polynomial denoting with $G(x)$  

$\text{Use consistent only notation } g(x)$  

$\text{Use of inconsistent notations}$  |

Overall, prospective teachers studied understand mathematics as a language consisting of symbols, graphics, definitions, and interrelationships among them. Each symbol has a certain meaning. This states that the language or terms used in mathematics can be translated into a particular sign [32]. Prospective teachers assume the language of mathematics is very influential in learning. Explanation of mathematical concepts to students must be done using the simple language understood by students. This is very important to note because when teachers use mathematical language that is not appropriate then students will not be able to explain mathematical ideas and concepts in appropriate language [33, 38]. Students have difficulty using mathematical terms and related concepts so learning must be able to be explained in simple languages so that students can more easily understand [6].

Prospective teachers understand the material being taught well. But this does not guarantee the teacher is also able to explain it well to students. [13] argues that although the teacher claims to have mastered the concept, the teacher is still confused in explaining mathematical concepts with appropriate mathematical language and is understood by students. This is indicated by the existence of some errors in the mathematical language of the prospective teachers studied.

Some of the mathematical errors of the prospective teachers are viewed from the semiotic aspects and aspects of grammatical patterns. Errors based on the semiotic aspects include errors in the use of symbols,
as well as inconsistent use of symbols and error writing operations symbols. While errors based on aspects of grammatical patterns include implicit logical relationships that are less precise and systematic in the application of concepts, as well as the use of vocabulary that can confuse.

In learning activities prospective teachers solve problems using mathematical symbols [14]. The most common mistakes made by prospective teachers are the use of symbols. Besides, the most frequent mistake made by prospective teachers is the mistake in mentioning the terms "minus vs negative" and "plus vs positive". In addition to the teacher’s weak awareness of the meaning of the concepts "minus vs negative" and "plus vs positive", the teacher is also still weak in the meaning of the concept of "simplify vs reduce", "similarity vs congruence" which causes misinterpretations of interpretations [33, 38].

Teacher candidates are sometimes inconsistent in the use of symbols. Besides, the implicit logical relationship of related sentences is also not well presented. Both of these are mistakes of prospective teachers in understanding the concept [18, 38] which must not happen. Prospective teachers need to be sure to master the language of mathematics well. Some mathematical concepts are difficult for students to understand so prospective teachers need to use mother tongue to understand students. It is very dangerous if the mother tongue used will make students misunderstand [38]. Therefore, teachers and prospective teachers need to explore the language of mathematics.

The solution to these problems is that teacher training needs to be focused on the use of appropriate mathematical language to help teachers make effective learning [18, 19, 38]. This is because the ability to communicate effectively through mathematical language requires mathematical understanding, vocabulary, and a strong knowledge base; flexibility, fluency, and ability to numbers, symbols, diagrams, and understanding skills [1, 38].

CONCLUSIONS

Understanding the mathematical language of prospective teachers with high and moderate IP is classified as very good while the understanding of the mathematical language of prospective teachers with low IP is classified as good. Prospective teachers have been able to understand some semiotic lists precisely such as sigma symbols and have also been able to present visual displays well. Some grammatical patterns in the form of vocabulary and logical relationships can also be used appropriately. But there are also some mistakes in understanding the language of prospective teacher mathematics. Errors in the semiotic aspects are errors in the use of symbols, inconsistent use of symbols, and writing errors. Mistakes based on aspects of grammatical patterns are implicit relationships that are less precise and systematic in the application of concepts, as well as the use of vocabulary that can confuse.

Understanding the prospective teacher’s mathematical language is influenced by learning achievement. This research focuses on understanding language from the aspect of professional competence, not on pedagogical competence with the subject of TPEP participants in positions (already a teacher). Therefore, it is necessary to further study the pedagogical competence of TPEP participants, both in positions and pre-positions (not yet becoming teachers).

REFERENCES


